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WOOLWICH MATHEMATICAL PAPERS

FOR ADMISSION INTO
THE ROYAL MILITARY ACADEMY

FOR THE YEARS
1885—1894

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MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JULY, 1885.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.]

1. If two triangles have two angles of the one equal to two angles of the other, each to each, and one side equal to one side, viz., sides which are opposite to equal angles in each; then shall the other sides be equal, each to each, and also the third angle of the one to the third angle of the other.

$ABCD$ is a rectangle, and AE , BF are drawn to meet the diagonals BD , AC in E and F respectively, and in such a direction that the angles AEB , AFB are equal to one another. Show that the triangles AEB , AFB are equal in all respects.

2. The opposite sides and angles of a parallelogram are equal to one another, and the diameter bisects the parallelogram, that is, divides it into two equal parts.

AB , CD , EF are three parallel straight lines: and the points A , C , E are in a straight line, and also the points B , D , F . Prove that if AC is equal to CE , then BD is equal to DF .

3. If a straight line be bisected and produced to any point, the rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

4. In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

What is the value in similar terms of the square on the side subtending one of the acute angles of this triangle?

$ABCDE$ is a straight line so divided that $AB = BC = CD = DE$, and O is an external point; show that the difference of the squares on OA and OE is twice the difference of the squares on OB and OD .

5. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

If any eight-sided figure be inscribed in a circle, show that the sum of either four alternate angles is equal to six right angles.

6. If from a point without a circle two straight lines be drawn, one of which cuts the circle, and the other touches it; the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square on the line which touches it.

Two equal circles intersect in A and B ; show that if from an external point O a tangent be drawn to each circle, the tangents will be unequal unless O lies on AB produced.

7. Inscribe a circle in a given triangle.

Find that straight line which would, if produced, bisect the angle between two given straight lines, without producing the given straight lines to meet.

8. Describe an isosceles triangle having either of the angles at the base double of the third angle.

9. What is Euclid's test for *Proportion*?

Triangles and parallelograms of the same altitude are to one another as their bases.

10. In a right-angled triangle, if a perpendicular be drawn from the right angle to the base, the triangles on each side of it are similar to the whole triangle, and to one another.

If AD be the perpendicular, and AB, AC the two sides including the right angle : prove that

$$\frac{1}{AD^2} = \frac{1}{AB^2} + \frac{1}{AC^2}.$$

11. Similar triangles are to one another in the duplicate ratio of their homologous sides.

II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Simplify
$$\frac{(423 \div \frac{3}{8}) + (1\frac{4}{5} \div 3) + (4\frac{1}{2} \div 1\frac{2}{3})}{(\frac{3}{7} \times 41) + (32 \times \frac{2}{3}) - (1 \div \frac{7}{10})}.$$

2. Express the value of $\frac{6.60416}{33.02083} \div \frac{2.772}{11.09}$ as a decimal.

3. A cube has an edge 2 ft. 6 in. long : find the ratio between the sum of the areas of the semicircles described on its edges, and the whole surface of the cube, having given that the area of a circle = 3.1416 times the square of the radius.

4. Extract the square root of $.02010724$, and the cube root of 18.609625 .

5. Find the value of $.54$ of $8s. 3d.$ + $.027$ of $\pounds 2. 15s. + 3125$ of $\pounds 2. 2s.$

6. If 24 oxen require 6 acres of turnips to supply them for 10 weeks, how many acres would supply 6 score of sheep for 15 weeks, 3 oxen eating as much as 10 sheep?

7. What weight must be added to $\frac{3}{8}$ of $\frac{5}{4}$ of half a cwt. to make it up to $\frac{3}{16}$ of $3\frac{3}{4}$ quarters avoirdupois?

8. Reduce $15s. 9\frac{3}{4}d.$ to the decimal of $\pounds 1$, and $\pounds 5. 7s. 6\frac{3}{4}d.$ to the decimal of one shilling.

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9. Define a common measure and a common multiple, and show how to find the Least Common Multiple of two numbers. Find the least number which contains the numbers 12, 20, 42, and 56.

10. If 3 tons, 13 cwts. 3 qrs. 12 lbs. cost £8. 12s. 4d., how much will 1 ton, 15 $\frac{3}{4}$ cwts. cost?

11. (a) How many superficial feet of inch plank can be sawn out of a log of timber 20 ft. 7 in. long, 1 ft. 10 in. wide, and 1 ft. 8 in. deep? Find this by duodecimals. (b) Show that an acre contains 10 square chains, assuming that 1 chain = 100 links, that 1 link = 0.66 of a foot, and that 640 acres = 1 square mile.

12. Find, by practice, the cost of making a road 29 m. 7 f. 200 yds. at £34. 16s. 8d. per mile.

13. Find the difference between the simple and compound interest on £378. 15s. for 2 years at 3 $\frac{1}{2}$ per cent.

14. Find the rate of interest when the discount on £226. 2s. 8d. due at the end of 1 $\frac{1}{2}$ years is £12. 16s.

15. A person has a sum of money invested in Consols (3 per cent.). He sells out at 87 $\frac{1}{2}$, and invests the proceeds in railway shares, when £100 sells for £174 $\frac{1}{2}$. He thus increases his income from £120 to £200. What dividend per cent. does the railway pay?

16. An open tank measuring on the outside 6 yds. 2 ft. in length; 3 yds. 2 ft. in width, and 7 ft. deep, is made with sides and floor of brick 1 ft. thick, and is filled with water. Find the weight of the tank and its contents, it being given that one cubic foot of water weighs 1000 oz., and that brick is one and a half times as heavy as water.

17. An astronomical clock has its dial divided into 24 divisions instead of 12, and the small hand goes round once in 24 hours, the large hand going round once every hour. The 24 hour is noon. Find when the hands are at right angles to one another between 24 and 1, and find also the interval between two successive meetings of the hands.

18. A legacy of £2420 is left to three persons in such proportions that after the payment of a legacy duty of 15 per cent. the first receives twice as much as the second, the second three times as much as the third. What are their respective shares?

19. What does a person gain or lose per cent. by selling butter at 8 $\frac{1}{2}$ d. per pound which cost £5. 5s. per cwt.?

20. A sum of £385. 17s. 6d. is due at the end of two years. What is its present worth, reckoning compound interest at 5 per cent.?

21. Find the value of $\log_{10} .01$, $\log_7 343$, $\log \sqrt[6]{8\sqrt{3}}$, and $2^{\frac{10}{\sqrt{66}}}$.

Given

$$\log 2 = .30103.$$

$$\log 3 = .4771213.$$

$$\log 11 = 1.0413927.$$

$$\log 3.0407 = .4829736.$$

$$\log 3.0408 = .4829879.$$

22. State and explain the rule for equation of payments.

A has to pay £100 per annum half-yearly for two years: find the equated time.

III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—Great importance will be attached to accuracy.]

1. Multiply $\frac{3x^3}{2} + 3x^2 - \frac{5x}{4} - \frac{23}{2}$ by $\frac{x^2}{6} - \frac{x}{3} + \frac{1}{2}$, and divide $32x + y^{\frac{5}{2}}$ by $2x^{\frac{1}{5}} + y^{\frac{1}{2}}$.

2. Find the highest common factor of

$$2x^3 - x^2 - x - 3 \text{ and } x^5 - x^3 - 4x^2 - 3x - 2.$$

and write down the L.C.M. of

$$9x^2 - 4, 4x^2 - 36, 3x^2 - 7x - 6, 3x^2 + 7x - 6.$$

3. Find the value of $x^3 + 2y^3 + 2z^3 + 6xyz$ when $x = y + z = \sqrt[3]{4}$, and if $\frac{x^2}{y^2} + \frac{y^2}{x^2} = 3$, prove that the value of $\frac{x^3}{y^3} - \frac{y^3}{x^3}$ will be 4 or -4.

4. Simplify

$$(1) \frac{1}{x-1} + \frac{2x+1}{x^2+x+1} - \frac{3}{x}.$$

$$(2) \left(\frac{x}{x-2} + \frac{5}{x-8} \right) \times \left(\frac{x-3}{3x-8} - \frac{2}{x+2} \right).$$

$$(3) \frac{(x-y)^4 - xy(x-y)^2 - 2x^2y^2}{(x-y)(x^3-y^3) + 2x^2y^2}.$$

5. Extract the square root of $47 - 12\sqrt{15}$, and find the value, when $x = \sqrt{3}$, of the expression

$$\frac{2x-1}{(x-1)^2} - \frac{2x+1}{(x+1)^2}.$$

6. Solve the equations

$$(1) \quad \frac{x - \frac{1}{2}}{x - 1} - \frac{3}{5} \left(\frac{1}{x - 1} - \frac{1}{3} \right) = \frac{23}{10(x - 1)}.$$

$$(2) \quad ax - by = 2ab, \quad 2bx + 2ay = 3b^2 - a^2.$$

$$(3) \quad y^2 + xy = 4, \quad x^2 + 2y^2 - xy = 8.$$

7. The perimeter of a right-angled triangle is six times as long as the shortest side. What is the ratio of the two sides containing the right angle?

8. Prove that p is the sum and q the product of the roots of the equation $x^2 - px + q = 0$.

Form an equation whose roots shall be the square of the sum, and the square of the difference of the roots of the equation $3x^2 + 3ax + a^2 = 0$.

9. The time during which a body will slide down a smooth inclined plane varies directly as the length and inversely as the square root of the vertical height of the plane. If the time of descent is one second when the height is 4 feet and length 8 feet, what is the height of a plane 1 yard long, down which a body will slide in half a second?

10. The 1st and 2nd terms of an harmonic series are 5 and 3 respectively. Find the next five terms.

A Geometric series, whose common ratio is $\frac{1}{n}$, has the same 1st and 2nd terms as an Harmonic series. Prove that the 3rd term of the former series will be equal to the $\frac{n+2}{2}$ th of the latter.

11. How many different sets of 12 can be chosen from a group of 15 men?

If 12 be selected from each of two groups in how many different ways can the 24 men be arranged in a line so that 4 men, and not more than 4 men from the same group, shall always stand together?

12. In what scale of notation will the decimal fraction .1 be expressed by .02222.....?

13. Write down the general term in the expansion of $(a - x)^{2n}$, and show that terms equidistant from the beginning and the end have the same numerical coefficient.

Also find the 11th term in the expansion of

$$\left(4x - \frac{1}{2\sqrt{x}} \right)^{15}.$$

IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy.]

1. Explain the different methods of measuring an angle by means of degrees and circular measure, and write down the relations between these two measures of an angle.

Find the number of degrees, and the circular measure of an angle of a regular octagon.

2. Define the cotangent and cosecant of an angle, and prove by geometrical constructions the formulae:

$$\begin{aligned}\operatorname{cosec}(180^\circ - A) &= \operatorname{cosec} A, & \cot(180^\circ - A) &= -\cot A, \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A.\end{aligned}$$

3. Find a general expression for all the angles which have the same tangent as a given angle.

Solve the equation:

$$\tan^2 \theta - (1 + \sqrt{3}) \tan \theta + \sqrt{3} = 0.$$

4. If $A + B$ be less than a right angle, prove by means of geometrical figures the formulae,

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B, \\ \sin A + \sin B &= 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B).\end{aligned}$$

5. Prove that

$$(1) \quad \tan \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}.$$

$$(2) \quad \frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4 \cos 2\theta \cos 4\theta.$$

$$(3) \quad \sin^{-1} \frac{3}{5} + \sin^{-1} \frac{8}{17} = \sin^{-1} \frac{77}{85}.$$

6. Prove that

$$\sin \frac{A}{2} + \cos \frac{A}{2} = \pm \sqrt{1 + \sin A},$$

and

$$\sin \frac{A}{2} - \cos \frac{A}{2} = \pm \sqrt{1 - \sin A},$$

and find the proper signs when the angle A lies between 90° and 180° .

Show by a geometrical figure that $\sin \frac{A}{2}$, when expressed in terms of $\sin A$, has four values.

7. Prove that in a triangle the sides are proportional to the sines of the opposite angles.

If, in a triangle ABC , $a \cos A = b \cos B$, prove that the triangle is isosceles or right angled.

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8. The lengths of two of the sides of a triangle are 1 foot and $\sqrt{2}$ feet respectively, and the angle opposite to the shorter side is 30° . Prove that there are two triangles which satisfy these conditions; find their angles, and show that their areas are in the ratio $\sqrt{3} + 1 : \sqrt{3} - 1$.

9. Prove that, in a triangle ABC

$$\cot \frac{C}{2} = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}.$$

If $a = 32$, $b = 40$, $c = 66$, find the angle C , having given

$$\log 207 = 2.3159703,$$

$$\log 1073 = 3.0305997,$$

$$L \cot 66^\circ 18' = 9.6424342,$$

and that the difference for $1'$ is 3431.

10. A ship sailing due north observes two lighthouses bearing respectively N.E. and N.N.E. After sailing 20 miles the lighthouses are seen to be in a line due east,

Having given

$$\log 2 = .3010300,$$

$$\log 11.715 = 1.0687423,$$

$$L \tan 22^\circ 30' = 9.6172243,$$

$$\log 11.716 = 1.0687794,$$

find the distance in miles between the lighthouses accurately to four places of decimals.

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[Full marks may be gained by doing about three-fourths of this paper. Great importance will be attached to accuracy in results.]

1. Draw a straight line perpendicular to a given plane from a given point without it.

What is the locus of the points which are equidistant from two given planes?

2. If two straight lines be cut by parallel planes, they shall be cut in the same ratio.

If a tetrahedron be cut by a plane parallel to two edges which do not meet, prove that the section is a parallelogram, and that its area is greatest when it bisects the other edges.

3. Define a pyramid and a regular pyramid.

How many regular pyramids can be constructed, the faces of which are equal equilateral triangles?

4. Prove that the section of a right circular cylinder by a plane is either a circle, an ellipse, or two parallel straight lines. If the section be an ellipse, prove that its foci are the points of contact of the two spheres which can be inscribed in the cylinder so as to touch the plane of the section.

5. If PN be the ordinate of a point P of a parabola, of which A is the vertex, and S the focus, prove that

$$PN^2 = 4AS \cdot AN.$$

If QQ' be a double ordinate of a parabola, and if PD be a straight line drawn parallel to the axis from a point P of the curve, and meeting QQ' in D , prove that

$$QD, DQ' = 4AS \cdot PD.$$

6. If the tangent at a point P of an ellipse meet the axis major in T and the axis minor in t , prove that

$$CN \cdot CT = AC^2, \text{ and } PN \cdot Ct = BC^2.$$

7. There are two equal fixed circles in a plane, which do not intersect each other. Prove that the locus of the centres of the circles which can be drawn touching the two circles consists of a straight line and a hyperbola, the transverse axis of which is equal to the diameter of either circle, and that its eccentricity is the ratio of the distance between the centres to the diameter.

8. If $(x, y), (x', y')$ be the coordinates of two points, find an expression for the distance between the points (1) when the axes are rectangular, (2) when the axes are inclined to each other at the angle ω .

Find the position of the centre of the circle circumscribing the triangle, the coordinates of whose angular points are $(2, 3), (3, 4), (6, 8)$, the axes being rectangular.

9. Find the polar equation of a circle, the pole being a point on the circumference. O is one of the points of intersection of two fixed circles, and OPQ is a straight line through O , meeting one circle in P and the other in Q ; find the locus of the middle point of the line PQ .

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10. Find the equation of the normal at any point of a parabola, and show that it can be put into the form

$$y = mx - 2am - am^3.$$

11. Define the eccentric angle at a point of an ellipse, and write down the equation of the tangent at the point in terms of the eccentric angle.

If CP , CD be conjugate semi-diameters of an ellipse, and if ϕ be the eccentric angle of the point P , find the equation of PD .

If ψ be the inclination of PD to the major axis, prove that

$$a \tan \psi = b \tan \left(\phi - \frac{\pi}{4} \right).$$

12. Trace the curve represented by the equation $x^2 - y^2 = a^2$, and find the value of c in order that the line, $y = mx + c$, may be a tangent to the curve.

Prove that the locus of the point of intersection of tangents to the curve, which are inclined to each other at the angle of 45° , is the curve

$$(x^2 + y^2)^2 = 4a^2(a^2 + y^2 - x^2).$$

13. Find the condition that the general equation of the second degree may represent a parabola, and interpret the equation

$$\sqrt{x} + \sqrt{y} = \sqrt{a}.$$

Find the locus of the middle points of the chords of this curve which are parallel to the line $x \sin \theta = y \cos \theta$.

VI. PURE MATHEMATICS (2).

[Great importance will be attached to accuracy in results.]

1. Find the coefficient of the middle term of $(1 + x + x^2)^n$, and prove that if each term of the coefficients be squared, the result will be the coefficient of the middle term of $(1 + x + x^2)^{2n}$.

2. Show that if m be any positive integer,

$$\left(1 + \frac{x}{m}\right)^m \text{ is } < \left(1 + \frac{x}{m+1}\right)^{m+1}, \text{ and that } 2^m - 1 \text{ is } > m \cdot 2^{\frac{m-1}{2}}.$$

3. If n Arithmetic and n Harmonic means be inserted between a and b , and a series of n terms be formed by dividing each Arithmetic by the corresponding Harmonic mean, prove that the sum of the series is

$$n \left\{ 1 + \frac{n+2}{n+1} \frac{(a-b)^2}{6ab} \right\}.$$

4. Prove that every convergent is nearer to the continued fraction than any of the preceding convergents.

If $\frac{P}{Q}, \frac{P'}{Q'}$ have the same first convergents $\frac{p_1}{q_1}, \dots, \frac{p_n}{q_n}$, the difference

$$\frac{P}{Q} - \frac{P'}{Q'} \text{ is } < \frac{1}{q_n^2}.$$

5. Define, and find the sum of n terms of a recurring series.

Sum the series $x + 3x^2 + 5x^3 + \text{etc.}$, to n terms.

6. Prove that

$$(1) \sin^4 \theta + 2 \cos \alpha \sin^2 \theta \cos^2 \theta + \cos^4 \theta = 1 - \left\{ \sin \frac{\alpha}{2} \sin 2\theta \right\}^2.$$

$$(2) \text{ If } u = \cot^{-1} \sqrt{\cos \alpha} - \tan^{-1} \sqrt{\cos \alpha}, \text{ prove that } \sin u = \tan^2 \frac{\alpha}{2},$$

and eliminate θ from the equations

$$x = a \cos \theta + b \cos 2\theta,$$

$$y = a \sin \theta + b \sin 2\theta.$$

7. Expand the sine and cosine of an angle in terms of its circular measure. If α, β, γ are the angles of a triangle, and a, b, c the sides opposite to them, and γ nearly $= \pi$, prove that $a + b = \frac{c}{1 + a\beta}$ nearly.

8. Expand θ in powers of $\tan \theta$, and hence derive a rapidly converging series for determining the value of π .

9. If $\cos p\theta + \cos q\theta = 0$, prove that the different values of θ form two Arithmetical Progressions in which the common differences are $\frac{2\pi}{p+q}$ and $\frac{2\pi}{p-q}$ respectively.

10. State the relations between the coefficients and the roots of an algebraic equation. If the roots of the equation $x^3 + px^2 + qx + r = 0$ be a, b , and c , form the equation of which the roots are $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$.

11. Two roots of the following equation are equal, find all the roots :

$$x^3 - 7x^2 + 16x - 12 = 0.$$

12. Prove that the numerically greatest negative coefficient increased by unity is a superior limit of the positive roots of an equation which is in its simplest form.

VII. PURE MATHEMATICS (3).

[Full marks will be given for correct answers to about two-thirds of this paper. Great importance will be attached to accuracy in results.]

1. Differentiate $\left(\frac{1-x}{1+x}\right)^n$.

2. Prove the formula for differentiating $y = uv$, where u, v are given functions of x : and apply it to find the differential coefficient of x^n when n is an integer,

3. Find the n th differential coefficient of $\log x$.

4. Expand $\log(1+e^x)$ by powers of x as far as x^2 .

5. How may it be ascertained whether a given function $f(x)$ continually increases, as x increases from a given value a to a given value b ?

Show that the function

$$x \sin x + \cos x + \cos^2 x$$

continually diminishes as x increases from 0 to 90° .

6. Find the least possible value of $\frac{\sqrt{a^2x^2 + b^2c^2}}{x+c}$.

7. Trace the curve $y = x - x^3$. Has it a point of inflexion? If a line be drawn from the origin to an infinitely distant point on the curve, what is the direction of this line?

8. Find the radius of curvature of the curve

$$y = x - \sin x$$

at the origin; also at the point where $x = \frac{\pi}{2}$.

9. Find the equation of the tangent at a given point on the curve

$$x^4 + y^4 = a^4.$$

What relation will hold between a, β , the portions intercepted on the axes by this tangent, whatever be the point of contact?

10. Integrate $\frac{dx}{1+x+x^2}$; $\cos^2 x dx$; $e^{ax} dx$.

11. Integrate $\sin^{-1} x dx$.

12. Reduce the integral $\int x^n \cos x dx$ to the form $\int x^{n-2} \cos x dx$.

13. Find the limit of the series

$$\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots, \frac{1}{2n}$$

when n increases indefinitely.

14. Show that in the curve

$$2y = e^x + e^{-x},$$

the area between the ordinates at any two points A, B varies as the arc AB .

VIII. STATICS.

[Great importance will be attached to accuracy.]

1. State the proposition known as the parallelogram of forces. If the forces are incommensurable, give a demonstration to prove the proposition so far as the *direction* of the resultant is concerned.

A uniform plane lamina in the form of a rhombus, one of whose angles is 120° , is supported by two forces applied at the centre in the direction of the diagonals, so that one side of the rhombus is horizontal; show that if P and Q be the forces, P being the greater, $P^2 = 3Q^2$.

2. If any number of forces acting on a particle can be represented in magnitude and direction by the sides of a polygon taken in order they will keep the particle in equilibrium.

Show that if a particle, placed in the centre of a regular polygon, be acted on by forces represented by the lines drawn from the particle to each of the angles it will be at rest.

3. Any system of forces acting in one plane on a rigid body may in general be reduced to a single couple and a single force acting at a given point of the plane, the given point being also the extremity of the arm of the resultant couple.

A square is acted on by forces 2, 4, 6, 8, taken in order along its sides; find the resultant and the resultant couple of these forces, if the centre of the square be taken for the given point.

4. Show that the centre of gravity of the perimeter of a given triangle is the centre of the circle inscribed in the triangle formed by joining the middle points of the given triangle.

5. In a system of three movable pulleys in equilibrium, in which the strings are parallel and each is attached to the weight (usually called the 3rd system), find the tension of the string which passes over the fixed pulley, the weights of the pulleys, which are all equal, being taken into account.

If the weights of the pulleys are neglected and each string is attached to a bar which keeps horizontal, find the point in the bar from which the weight is suspended.

6. Give examples of stable and unstable equilibrium. A cone and a hemisphere of the same material are cemented together at their common circular base: if they are placed on a horizontal plane, the hemisphere being in contact with the plane, find the height of the cone that the equilibrium may be neutral, it being given that the centre of gravity of a hemisphere divides a radius in the ratio of 3 to 5.

7. If any number of forces act in one plane on a rigid body, *state* the three conditions of equilibrium.

A beam whose weight is (W) rests with its ends on two inclined planes whose angles of inclination are (α) and (β); prove that the sum of the pressures on the planes is

$$W \cdot \frac{\cos \frac{\alpha - \beta}{2}}{\cos \frac{\alpha + \beta}{2}}.$$

8. On what hypothesis is the relation between (P) and (W) usually obtained on a screw considered as a mechanical power? If the screw be rough, and (R) the sum of the normal resistances on the thread of the screw, (W) the weight supported, (ϕ) the angle of resistance, and (α) the pitch of the screw, obtain the equation $R = \frac{W \cos \phi}{\cos (\alpha + \phi)}$, where (P) is just on the point of prevailing over (W).

9. The same force acting parallel to two inclined planes of 30° and 60° inclination can just move a given weight up the plane of less inclination, and can just prevent a weight twice as great from moving down the plane of greater inclination; prove that the coefficient of friction which is the same for both planes is nearly $\frac{1}{3}$.

10. State the principle of "virtual velocities." If a weight (W) be supported in equilibrium on a smooth inclined plane by a weight (P) hanging vertically and passing over a fixed pulley placed in the prolongation of the height of the plane, prove that the principle of virtual velocities holds good, and show that for a small displacement the centre of gravity of (P) and (W) will neither ascend nor descend.

11. A heavy ladder is placed in a given position, between a vertical wall and the horizontal ground, both being considered equally rough; a workman of given weight ascends the ladder with a given load, show how to determine by a *geometrical construction* whether the ladder will slip.

IX. DYNAMICS.

[Great importance will be attached to accuracy.]

1. The measure of a certain velocity in feet per second is v ; what is it in miles per hour?

2. Two trains, each 200 feet long, are moving towards each other with velocities of 20 and 30 miles per hour respectively. Find the time which elapses from the instant when they first meet till they have completely cleared each other.

3. State exactly what is meant by saying that the accelerating force of gravity is 32.

In the equation $F = mf$, if a foot and a second are the units of space and time, and a weight of one pound the unit of force, what is the unit of mass?

4. Two bodies which weigh 9 lbs. and 16 lbs. respectively at the earth's surface, are placed in space at a distance asunder of 100 feet, no force acting on either. If they were now to attract each other with a constant force equal to 1 lb. at all distances, find after what time they would meet.

5. A body slides down a rough inclined plane 100 feet long, the sine of whose inclination = 0.6, and coefficient of friction = $\frac{1}{2}$: find its velocity at the bottom.

If projected up the plane with a velocity which just carries it to the top, find what height it would reach if thrown vertically upwards with that velocity.

6. Two perfectly elastic balls whose masses are M , M' , moving in the same direction, strike each other. If the hindmost ball is reduced to rest by the blow, show that its velocity must have been more than double that of the other.

WOOLWICH ENTRANCE EXAMINATION.

7. A train runs from rest down an incline of 1 in 100 for a distance of 1 mile (no engine attached): it then runs up an equal gradient with its acquired velocity for a distance of 500 yards before stopping. Assuming the principle of work, find the total resistance, frictional or other, in pounds per ton, which has been opposing its motion.

8. A body is projected horizontally with a given velocity. Prove that it describes a parabola, and determine the position of its focus.

9. Two bodies are projected from the same point, one later than the other by T seconds, so as to describe the same parabola. Show that they are nearest to each other when in the same horizontal line (if that is possible) and that this occurs at the interval of time $\frac{V \sin \alpha}{g} - \frac{T}{2}$ after the second body was projected. Explain what circumstances as to the *data* are required, if this be possible.

10. Two weights, P , Q , are connected by a light string passing over a smooth fixed peg. Find the acceleration of the system. Mention any experimental use which has been made of this contrivance.

If two equal weights P , P , are in equilibrium, connected in this way, and a third weight, P , is laid on one of them, find by how much the pressure on the peg is increased.

11. A particle starting with a velocity u , falls down a smooth vertical curve of any form. State what its velocity is when it has arrived at any given point of the curve.

A particle falls down a vertical circle, starting from rest at the highest point. If, when at any point, its velocity be resolved into two components, one passing through the centre, the other through the lowest point of the circle, prove that the latter is of constant magnitude.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1885.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.]

1. Define a *right angle*, and show how to draw a straight line at right angles to a given straight line, from a given point in the same.

If one diagonal of a quadrilateral figure bisect the two angles at its extremities, it will bisect the other diagonal at right angles.

2. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are equal to two right angles.

The longer sides of a parallelogram are twice as long as the shorter sides. Show that the straight lines joining the middle point of one of the longer sides with the ends of the opposite side, are perpendicular to each other.

3. If a parallelogram and a triangle be on the same base and between the same parallels, the parallelogram shall be double of the triangle.

4. If a straight line be divided into two equal and also into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line.

WOOLWICH ENTRANCE EXAMINATION.

If any point D be taken in the base BC of an isosceles triangle ABC , the rectangle under BC , CD will be equal to the difference between the squares on AB , AD .

5. If any point be taken without a circle, and straight lines be drawn from it to the circumference, one of which passes through the centre, of those which fall on the concave circumference, the greatest is that which passes through the centre, and of the rest, that which is nearer to the one passing through the centre is always greater than one more remote; but of those which fall on the convex circumference, the least is that between the point without the circle and the diameter; and of the rest, that which is nearer to the least is always less than one more remote.

Prove that the two lines which are equally inclined to the shortest line are equal.

6. Define a *segment of a circle*, and show how to bisect its circumference.

Prove that the straight lines joining the ends of the base of a segment of a circle with any point on its circumference, are equally inclined to the straight line passing through that point and the point of bisection of the circumference.

7. About a given circle describe a triangle equiangular to a given triangle.

Prove that the equilateral triangle inscribed in a circle is one-fourth of the equilateral triangle described about the same circle.

8. Inscribe an equilateral and equiangular pentagon in a given circle.

9. The sides about the equal angles of triangles which are equiangular to one another are proportionals.

Prove that the diameters of the circles circumscribing the two triangles formed by joining the vertex with any point in the base of a given triangle are proportional to the sides of that triangle.

10. If four straight lines be proportionals the similar rectilineal figures similarly described on them shall also be proportionals; and if the similar rectilineal figures similarly described on four straight lines be proportionals, those straight lines shall be proportionals.

11. If the vertical angle of a triangle be bisected by a straight line which also cuts the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base, together with the square on the straight line which bisects the angle.

II. ARITHMETIC.

(Including the use of Common Logarithms.)

[Great importance will be attached to accuracy.]

1. Multiply five million forty-six thousand and one, by four thousand and eight ; and divide '0064 by '51863 to seven places of decimals.
2. When is the decimal equivalent to a vulgar fraction a terminating decimal, and when a circulating decimal? Multiply together, '4, '4, and '0004.
3. Find a mean proportional between 36 and 2401, and extract the cube root of 128'024064.
4. Find a servant's wages for five months three weeks six days at one pound seven shillings and five pence per month, reckoning seven days to a week, and four weeks to a month.
5. If 2 cwts. 2 qrs. of sugar cost £3. 10s. 0d., what will be the cost of 28 lbs. 4 ozs.?
6. Find how many flagstones, each 5'76 ft. long and 4'15 ft. wide, are requisite for paving a cloister which encloses a rectangular court 45'77 yds. long and 41'93 yds. wide : the cloister being 12'45 ft. wide.
7. If the carriage of 17 cwts. 3 qrs. for $7\frac{1}{2}$ miles cost £1. 0s. $8\frac{1}{2}$ d., what weight should be carried 20 miles for 16s. 4d.?
8. Reduce 1 r. 14 p. to the decimal of an acre ; and find the value of '05854 of a guinea.
9. What was the average price of a quantity of cattle, of which 60 were bought for £10. 18s. 6d. each, 60 for £10. 16s. 9d. each, 50 for £10. 15s. 3d. each, and 30 for £10. 12s. 6d. each?
10. In how many months will £1550. 12s. 6d. amount to £1580. 15s. $6\frac{1}{2}$ d. at $3\frac{1}{2}$ per cent. per annum simple interest?
11. What is the difference between the simple and compound interest on £1750 for 3 years, at 4 per cent. per annum?

WOOLWICH ENTRANCE EXAMINATION.

12. Explain the difference between true discount and banker's discount, and find the difference between the two for £610 due in 4 months at 5 per cent.

13. What was the cost price of an article which was sold for £1. 4s. 6d. and on which there was a profit of $16\frac{2}{3}$ per cent.?

14. Which of the following stocks is the best for investment, and which the worst: 3 per cents. at $83\frac{1}{2}$, $3\frac{1}{2}$ per cents. at $98\frac{7}{8}$, or 4 per cents. at $107\frac{1}{2}$? Find the yield of £100 in each.

15. If £1000 3 per cents. be sold at $91\frac{3}{4}$, and with the proceeds 5 per cent. railway stock be bought at 125, find the increase of income.

16. Find (by duodecimals) the cost of cultivating a garden 109 ft. 6 in. long and 58 ft. 6 in. wide, at $4\frac{3}{4}d.$ per square yard.

17. Two boats start for a race at 3 o'clock. The race is over at $6\frac{2}{3}$ minutes after 3, the losing boat being 40 yards behind at the finish: at 4 minutes past 3 this boat was 700 yards from the winning post. Find the length of the course, and the speed (supposed uniform) of each boat in miles per hour.

18. If 7 rix dollars are worth 2 ducats and 9 ducats worth 4 moidores, and 20 moidores worth £27: how many rix dollars are there in £72?

19. A cistern is 6 ft. long and 5 ft. broad, and it is of such a depth that it would take 1440 bricks, each 9 inches long, 3 inches deep, and 4 inches broad, to fill it: how many gallons would the cistern hold, if a pint of water contains 72 cubic inches?

20. Define a logarithm, and state the chief uses of logarithms.

Given $\log 86750 = 4.947519$, write down $\log 867.50$, $\log 8.6750$, $\log .086750$.

21. Given $\log 2 = .3010300$, $\log 3 = .4771213$, find $\log 9$, $\log 128$, $\log 2^{\frac{1}{100}}$, $\log 2.4$.

22. If the logarithm of a number to base 4 is .35184, what is its logarithm to base 8?

III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—Great importance will be attached to accuracy.]

1. Divide $2a^2 - \frac{55ab}{12} + \frac{29ac}{9} + \frac{21b^2}{8} - \frac{15bc}{4} + \frac{c^2}{3}$ by $\frac{2a}{3} - \frac{3b}{4} + c$.

2. Find the highest common factor of

$$4x^4 - 9x^2 + 6x - 1 \quad \text{and} \quad 6x^3 - 7x^2 + 1.$$

When of two algebraical expressions the H.C.F. has been found, what is the rule for determining their L.C.M.?

3. Simplify:

(i.) $\frac{b}{a+b} - \frac{ab}{(a+b)^2} - \frac{ab^2}{(a+b)^3};$

(ii.) $\left(\frac{x}{x-y} - \frac{y}{x+y}\right)(x^2 + 2xy - y^2) \div \left(\frac{x}{x-y} + \frac{y}{x+y}\right);$

(iii.) $\frac{a^4 + x^4 + ax(a^2 + x^2) + a^2x^2}{a^5 - x^5} \div \frac{a^2 + x^2 + ax}{a^3 - x^3}.$

4. Find the square root of

(i.) $\frac{9a^{2m}c^2}{4b^{12}} - \frac{3a^{m+n}c}{b^3} + a^{2n}b^6 - \frac{2^9a^m c}{b^6} + \frac{2^9a^n b^3}{3} + \frac{2^{16}}{9};$

(ii.) $87 - 12\sqrt{42}.$

5. Find the value of

(i.) $(2^{\frac{1}{2}}a + 3^{\frac{1}{2}}b)(3^{\frac{1}{2}}a - 2^{\frac{1}{2}}b) - 6^{\frac{1}{2}}(a^2 - b^2) + 2^{\frac{1}{2}}ab;$

(ii.) $(35\sqrt{10} + 77\sqrt{2} + 63\sqrt{3} + 28\sqrt{15}) \times (\sqrt{10} - \sqrt{2} - \sqrt{3}).$

6. Solve the equations

(i.) $\frac{3\{ab - x(a+b)\}}{a+b} + \frac{(2a+b)b^2x}{a(a+b)^2} = \frac{bx}{a} - \frac{a^2b^2}{(a+b)^3};$

(ii.) $\begin{cases} (x+5)(y+7) = (x+27)(y+\frac{5}{7}), \\ xy = 1; \end{cases}$

(iii.) $\begin{cases} y+z = 3xyz, \\ z+x = 2xyz, \\ x+y = \frac{8}{3}xyz. \end{cases}$

WOOLWICH ENTRANCE EXAMINATION.

7. Divide 11 into three parts so that the 1st may be to the 2nd as $\frac{1}{2}$ to $\frac{1}{3}$, and the 2nd to the 3rd as $\frac{1}{4}$ to $\frac{1}{5}$.

8. Show that p and q represent respectively the sum and product of the roots of the equation

$$x^2 - px + q = 0.$$

The area of an oblong room is 328 square feet, and its perimeter is 73 feet; write down and solve the quadratic equation which gives the lengths of the sides of the room.

9. If $a : b :: c : d$, prove that

$$4a^6 + 5b^6 : 4c^6 + 5d^6 :: a^3b^3 : c^3d^3.$$

10. Prove the rule for expressing a given integer in any proposed scale.

758·t83 cubic feet being the volume (expressed in the duodenary scale) of a cube; find the number of feet and inches in an edge of the cube.

11. If the first and last terms of a geometrical progression of 8 terms be respectively a^7 and β^7 ; find the sum of the series.

Prove the rule for finding the value of a recurring decimal in which all the figures recur.

12. Find the number of permutations (p_r) of n things, taken r at a time.

Show that

$$(p_3 - p_2)(p_4 - p_3) \dots (p_{n-1} - p_{n-2}) = \frac{p_2 p_3 \dots p_{n-1} p_n}{p_3 p_{n-1}}.$$

13. Assuming the Binomial Theorem when the exponent is a positive integer, prove it when the exponent is any positive quantity.

Find the 13th term in the expansion of

$$(2^6 + 2^6 x)^{\frac{11}{2}}.$$

IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy.]

1. Define the sine and cosine of an angle, and find the values of the sines and cosines of 30° , of 210° , and of 930° .

2. Trace the changes in sign and magnitude of the expressions

(1) $\cos x - \sin x$.

(2) $\sin(\pi \sin x)$, as x increases from 0 to 2π .

3. If $\cos x = \cos a$, find a general expression for all the values of x .

Solve the equations—

$$\cos(2x + 3y) = \frac{1}{2}, \quad \cos(3x + 2y) = \frac{\sqrt{3}}{2}.$$

4. Prove the formulae

(1) $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

(2) $\cos(A + B) \sin B - \cos(A + C) \sin C = \sin(A + B) \cos B - \sin(A + C) \cos C$.

(3) $4 \sin 20^\circ \sin 40^\circ \sin 80^\circ = \sin 60^\circ$.

5. Prove that in any triangle ABC ,

(1) $c = a \cos B + b \cos A$.

(2) $c^2 = a^2 + b^2 - 2ab \cos C$.

6. If R be the radius of the circle circumscribing a triangle ABC prove that

$$2R = \frac{a}{\sin A} = \frac{a + b + c}{\sin A + \sin B + \sin C}.$$

If O be the centre of this circle, and R_1 the radius of the circle circumscribing the triangle BOC , prove that

$$2R_1 \cos A = R.$$

7. Prove the formulae

(1) $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x + y}{1 - xy}$.

(2) $\tan^{-1} \frac{1}{3} + \sin^{-1} \frac{1}{\sqrt{5}} = 45^\circ$.

WOOLWICH ENTRANCE EXAMINATION.

8. If perpendiculars be drawn from the angular points of a triangle ABC on the opposite sides, prove that the lengths of them intercepted between the angular points and the orthocentre (*i.e.*, the point of intersection of the perpendiculars), are proportional respectively to $\cos A$, $\cos B$, and $\cos C$, and that the lengths of them intercepted between the orthocentre and the sides are proportional to $\sec A$, $\sec B$, and $\sec C$.

9. Prove that the area of a triangle ABC is equal to $\frac{1}{2}bc \sin A$.

If $B = 45^\circ$, $C = 60^\circ$, and if $a = 2(\sqrt{3} + 1)$ inches, prove that the area of the triangle is $6 + 2\sqrt{3}$ square inches.

10. From each of two ships a mile apart the angle is observed which is subtended by the other ship and a beacon on shore; these angles are found to be $52^\circ 25' 15''$ and $75^\circ 9' 30''$ respectively.

Having given

$$L \sin 75^\circ 9' 30'' = 9.9852635$$

$$L \sin 52^\circ 25' 15'' = 9.8990055$$

$$\log 1.2197 = .0862530$$

$$\log 1.2198 = .0862886,$$

find the distances of the beacon from each of the ships, one distance exact and the other to five places of decimals.

11. A man who is walking on a level plain towards a tower observes at a certain point that the elevation of the top of the tower is 10° , and, after going 50 yards nearer to the tower, that the elevation is 15° .

Having given

$$L \sin 15^\circ = 9.4129962,$$

$$\log 25.783 = 1.4113334,$$

$$L \cos 5^\circ = 9.9983442,$$

$$\log 25.784 = 1.4113503,$$

find the height of the tower in yards to four places of decimals.

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

*[Full marks may be gained by doing about three-fourths of this paper.
Great importance will be attached to accuracy in results.]*

1. Draw a straight line perpendicular to a given plane from a given point without it.

Find the locus of a point in a plane such that the straight lines which join it to two given points at equal altitudes above the plane may be equally inclined to the plane.

2. If two planes which cut one another are each of them perpendicular to a third plane, their line of intersection is also perpendicular to the same plane.

3. Assuming that pyramids with equal bases and of equal altitudes are equal to one another, prove that a triangular prism may be divided into three equal triangular pyramids.

Show that the altitude of a regular tetrahedron is to the length of each of its edges as $\sqrt{2} : \sqrt{3}$.

4. The curve formed by the intersection of the surface of a right circular cone with a plane, not passing through the vertex, is a parabola, if the inclination of the cutting plane to the axis of the cone is equal to the constant angle between the generating line and the axis.

5. Draw a pair of tangents to a given parabola from a given external point.

If O is the external point, P, Q the points of contact, and O' the angle opposite to O of the parallelogram constructed on OP, OQ : OO' is parallel to the axis of the parabola.

6. In an ellipse or hyperbola, the perpendiculars from the foci on the tangent intersect it in points which lie on the circumference of the circle described on the major or transverse axis.

Find the foci of an ellipse, whose major axis is a diameter of a given circle, and which touches a given chord of the circle at a given point in it.

WOOLWICH ENTRANCE EXAMINATION.

7. Find the length of the perpendicular from the origin upon the straight line which joins the points $(a \cos \alpha, a \sin \alpha)$, $(a \cos \beta, a \sin \beta)$, and show that the perpendicular bisects the distance between these points.

8. Find the equation of the tangent to the circle

$$x^2 + y^2 + Kx + Ly + M = 0,$$

at a given point (x', y') .

The equations of two circles are

$$x^2 + y^2 + Ax + By = 0, \quad x^2 + y^2 + Cx + Dy = 0.$$

Find the locus of the point of intersection of the circles, other than the origin, when $A - C$, $B - D$ vary proportionally.

9. Find the polar equation of the tangent to a parabola at a given point, the focus being the pole.

If the tangents at P , Q to a parabola of which the focus is S meet in T , show that $ST^2 = SP \cdot SQ$.

10. Express the focal distances of any point on an ellipse, in terms of the abscissa; and if the normal at P meets the axis major in G , show that

$$PG^2 = \frac{b^2}{a^2} r r'; \quad a, b \text{ being the semi-axes; } r, r' \text{ the focal distances of } P.$$

11. A hyperbola being defined by the equations $x = a \sec \theta$, $y = b \tan \theta$, show that if 2α and 2β are the sum and difference of the values of θ corresponding to two points on the curve, the equation of the chord joining the points is $a^{-1} \cos \beta \cdot x - b^{-1} \sin \alpha \cdot y = \cos \alpha$.

Prove that a chord which joins the extremities of a pair of conjugate diameters of a hyperbola is parallel to one of the asymptotes.

VI. PURE MATHEMATICS (2).

[Great importance will be attached to accuracy.]

1. Expand $\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$ according to the powers of x .

2. Find the sum of the products, two and two, of every pair of the natural numbers 1, 2, 3 ... n .

3. In how many ways may n prizes be distributed among n boys, if every boy is to have a prize?

Prove that if $n+1$ prizes be given away to n boys, and every boy is to have at least one, the number of ways in which they may be given is

$$\frac{1}{2}n(n+1)/n.$$

4. Find x, y, z from the equations

$$x + ya + z\beta + 2 = 0,$$

$$x + ya^2 + z\beta^2 + 3 = 0,$$

$$x + ya^3 + z\beta^3 + 5 = 0,$$

where a, β are the roots of the equation $u^2 = u + 1$.

5. How is the logarithm of a number to any base, a , to be found from an ordinary table of logarithms?

If a number $a + \delta$ exceed the base a by a very small quantity δ , prove that its logarithm is nearly $1 + \frac{\delta}{a \log_e a}$.

6. Given one root, a , of the equation $x^4 + 1 = 0$, express the three remaining roots in terms of a .

7. If α, β, γ be the roots of the cubic $x^3 + ax^2 + bx + c = 0$, find the value of the product

$$(\alpha^2 + \beta^2)(\alpha^2 + \gamma^2)(\beta^2 + \gamma^2).$$

8. State Des Cartes' rule of signs. What inference may be deduced from it as to the roots of the equation $x^3 + 2x^2 + 3 = 0$?

9. If $2 \cos a = x + \frac{1}{x}$, prove that $2 \cos ra = x^r + \frac{1}{x^r}$.

10. Through a fixed point O within a circle any chord AOB is drawn: find the relation between the angles which AO, OB subtend at the centre.

11. What value should be given to the constant m in order that the formula

$$a = \frac{\tan a + m \sin a}{1 + m}$$

shall be the best approximation to the circular measure of a small angle a , in terms of its sine and tangent?

12. Find a symmetrical relation between the cosines of three angles, A, B, C whose sum is 180° .

VII. PURE MATHEMATICS (3).

[Full marks may be obtained for about two-thirds of this paper. Great importance will be attached to accuracy in results.]

1. Differentiate with regard to x the expressions :

$$\frac{2x}{1-x^2}, \tan^{-1} \frac{2x}{1-x^2}, \text{ and } \log \frac{4x^2}{(1-x^2)^2}.$$

2. If $y = e^{3x} \cos 4x$, find $\frac{dy}{dx}$, and prove that, if $\tan \phi = \frac{4}{3}$,

$$\frac{d^2y}{dx^2} = 5e^{3x} \cos(4x + \phi).$$

Find the general term in the expansion, by Maclaurin's theorem, of $e^{3x} \cos 4x$ in powers of x .

3. Find the maximum value and the minimum value of the expression,

$$x^3 - 12x + 24.$$

Find also the values of x which make the expression,

$$(2 - 2x) \sin a - \cos(2x - a),$$

a maximum or a minimum.

4. Find the equations of the tangent and normal at the point (x, y) of the curve

$$x^3 + y^3 = 2c^3.$$

If α and β be the distances from the origin of the points where the tangent cuts the axes of x and y , prove that

$$\alpha^{-\frac{3}{2}} + \beta^{-\frac{3}{2}} = 2^{-\frac{1}{2}} c^{-\frac{3}{2}}$$

Prove also that the normal at the point (c, c) passes through the origin.

5. Prove that, at a point of inflexion on a curve, $\frac{d^2y}{dx^2}$ changes sign.

Find the points of inflexion of the curve

$$y = \frac{x^2}{2} - \frac{9}{10} (x - 1)^{\frac{5}{3}}.$$

6. If ϕ be the angle between the radius vector to any point of a curve, $r = f(\theta)$, and the tangent at the point, prove that $\tan \phi = \frac{r d\theta}{dr}$.

If the curve be $r = a(1 - \cos \theta)$, prove that $2\phi = \theta$, and that, if p be the perpendicular from the origin upon the tangent, $r^3 = 2ap^2$.

7. Trace the curves

$$(1) \ y = (x-1)(x^2-4);$$

$$(2) \ r = a \sin 3\theta.$$

8. Find the radius of curvature of the curve

$$ay^2 = x^3$$

at the origin, and also at the point (a, a) .

9. Integrate with regard to x the expressions

$$(1) \ \frac{1}{x+1}, \quad (2) \ \frac{1}{(x+1)(x+2)}, \quad (3) \ \frac{1}{\sqrt{x^2+1}}.$$

10. Explain the method of integration by parts, and integrate with regard to x the expressions

$$e^x \cos x \text{ and } x \log(x+1).$$

11. Find the limiting value of the sum of the series

$$\frac{1}{n} \left\{ \sin \frac{\pi}{2n} + \sin \frac{2\pi}{2n} + \sin \frac{3\pi}{2n} + \dots + \sin \frac{(n-1)\pi}{2n} \right\}$$

when n is increased indefinitely.

12. Find the areas enclosed by the curves :

$$(1) \ ay = x\sqrt{a^2 - x^2}, \quad (2) \ r = a \sin 3\theta.$$

VIII. STATICS.

[Full marks may be gained by correctly answering three-quarters of this paper. Great importance is attached to accuracy.]

1. Three forces acting at a point make equilibrium. If they make angles of 120° with each other, prove that they are equal. If the angles are 60° , 150° , 150° , in what proportions are the forces?

2. Two given forces meet at a point. Find in what direction a third force of given magnitude must act at the point, if the resultant of the three is the greatest possible.

WOOLWICH ENTRANCE EXAMINATION.

3. *State* the conditions necessary for equilibrium for any system of forces acting in one plane on a rigid body.

If the rigid body have one point fixed, what is necessary for equilibrium? What, if it have two points fixed?

4. The door of a room is open : determine (as far as possible) the forces which it exerts upon its two hinges ; given all the dimensions, and its weight.

5. Two given forces act at two given points ; if they are turned round those points in the same direction through any two equal angles, show that their resultant will always pass through a fixed point.

Extend this theorem to any number of forces,

6. A uniform bar AB 10 feet long and weighing 50 lbs. rests on the ground. If a weight of 100 lbs. be laid on it at a point distant 3 feet from B , find what vertical force applied at the end A will just begin to lift that end.

7. Prove in any way that a couple cannot be balanced by a single force. If it be balanced by *two* forces, what is necessary with regard to them?

8. A heavy horizontal circular ring rests on three supports at the points A, B, C of its circumference. Given its weight, and the sides and angles of the triangle ABC , find the pressure on the supports.

9. Two inclined planes have a common vertex and equal slopes ; one is smooth and the other rough. A given weight W is supported on one by a string passing over the vertex and attached to another weight resting on the other plane ; this weight being just sufficient to prevent W descending. Prove that the weight required is less when W is on the rough plane, than when it is on the smooth one.

10. Four equal heavy bars are jointed so as to form a rhombus $ABCD$: A and C are joined by a string. The whole is suspended from the angle A ; find the tension of the string, by means of the principle of Virtual Velocities (or otherwise).

11. If the resultant of any systems of parallel forces in one plane acts outside the two extreme forces of the system, show that some must act in an opposite sense to others.

A heavy bar lies on a rough floor : a horizontal force, too small to move it, is applied at one end, at right angles to the bar ; describe *generally* the forces which have been called into operation to prevent motion ensuing,

12. A circle is pushed over an equal circle till it completely covers it. Find the centre of gravity of the crescent-shaped space left uncovered just before they coincide.

13. A heavy rectangular block $ABCD$ rests with AB on the ground; a rope is attached to the corner C and pulled round a fixed pulley P vertically over D till motion ensues. Give a *geometrical construction* determining the least height for the pulley P , if the block is to begin to revolve round A without slipping along the ground (given all particulars).

IX. DYNAMICS.

[Great importance will be attached to accuracy.]

[If needed, the measure of the force of gravity may be taken as 32 feet.]

1. A body moves in any direction in a vertical plane: given the horizontal and vertical velocities at any instant, show how to find the actual velocity of the body, and the direction in which it is moving.

A body describes uniformly a vertical circle, whose radius is 12 inches, in 6 seconds, and the centre of the circle moves uniformly in a horizontal line with the same velocity: show how to find the true velocity of the body whenever its direction is inclined at 45° to the horizon.

2. State the first law of motion, and the evidence on which it is accepted.

In an observed case of a body in uniform motion, are we to infer that no force acts on the body? Illustrate this by example.

3. How is uniform force measured? If a body be acted upon by a uniform force, and be initially projected in the direction of that force with a velocity (u), and if it acquire a velocity (v) in (t) seconds, prove that (s) the space described may be determined by the equation $s = t\left(u + \frac{v}{2}\right)$.

A train at the foot of an incline is moving at the rate of 60 miles an hour; the steam is suddenly turned off, and it then runs on the horizontal plane for $3\frac{1}{2}$ miles before it stops; if friction be the constant retarding force, find its measure. Determine also the space the train describes in three minutes from the instant the steam is turned off.

4. A body slides down a rough inclined plane, show that the acceleration is $g \frac{\sin \alpha - \phi}{\cos \phi}$ where (ϕ) is the limiting angle of resistance.

WOOLWICH ENTRANCE EXAMINATION.

If the height of the inclined plane be 12 feet, the base 16 feet, find how far a body will move on the horizontal plane after sliding down from rest the length of the inclined plane, supposing it to pass from one plane to the other without loss of velocity, the coefficient of friction for both planes being $\frac{1}{8}$.

5. In the case of a projectile in vacuo, find the direction of the projectile with respect to the horizon at any point of its course.

If (θ) be the angle of elevation at which a projectile strikes a given mark, (α) the angle of projection, and (45°) the angle which the given mark subtends at point of projection, prove $\tan \alpha + \tan \theta = 2$.

6. State the law of motion that connects the statical and dynamical measures of force. Obtain the equation from which that connexion is shown, and explain the units of reference in that equation.

A body weighing 36 lbs. is moved by a constant pressure, which generates in it a velocity of 8 feet per second, find the statical measure of that pressure.

7. A ball is projected vertically upwards with a velocity of 160 feet per second, when it has reached its greatest height it is met in direct impact by an equal ball which has fallen through 64 feet; find the times from the instant of impact in which the balls reach the ground, the elasticity between them being $\frac{1}{2}$.

8. Assuming the expression for the time of a small oscillation of a pendulum in a circular arc, calculate to two decimal places the length of a second's pendulum in inches.

A simple pendulum beating seconds is lengthened by $\frac{1}{20}$ of an inch; find the number of seconds it will lose in 24 hours.

9. A string 4 feet long can just support a weight of 9 lbs. without breaking, a weight of 8 lbs. fixed to one end of the string describes a circle uniformly round the other end, which is fixed on a smooth horizontal table; determine the greatest number of revolutions the revolving weight can make in a minute so as just not to break the string.

10. Determine the time of a small oscillation in a cycloid.

11. Show how to find the work accumulated in a moving body.

A bullet with an initial velocity of 1500 feet strikes a target 1200 yards distant with a velocity of 900 feet in a second, the range of the bullet being assumed to be horizontal; compare the mean resistance of the air with the weight of the bullet.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1886.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.]

1. Any two sides of a triangle are together greater than the third.

Prove that the shortest line, which can be drawn with its ends upon the circumferences of two concentric circles, will, when produced, pass through the centre.

2. Prove one case of the following proposition:—

If two triangles have two angles of the one equal to two angles of the other, each to each; and one side equal to one side, viz., either the sides adjacent to the equal angles in each, or sides which are opposite to equal angles in each; then shall the other sides be equal, each to each; and also the third angle of the one equal to the third angle of the other.

The vertical angle A of the isosceles triangle ABC is half a right angle, and the perpendiculars AD , BE let fall from A , B upon the opposite sides intersect in F . Show that FE is equal to EC .

3. Describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given angle.

4. If a straight line be divided into any two parts, the rectangle contained by the whole and one of the parts is equal to the rectangle contained by the two parts, together with the square on the aforesaid part.

5. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

Describe a right-angled triangle such that the rectangle contained by the hypotenuse and one of the sides containing the right angle may be equal to the square on the other side.

6. Find the centre of a given circle.

Describe a circle of given radius passing through a given point, and touching a given straight line.

When is this construction impossible?

7. In equal circles, the arcs on which equal angles stand are equal to one another, whether the angles be at the centres or circumferences.

The tangent at A to the circle circumscribing the triangle ABC cuts BC produced in D , and AC produced cuts the circumference of the circle circumscribing the triangle ABD in E . Prove that AD is equal to DE .

8. Inscribe an equilateral and equiangular hexagon in a given circle.

If $ABCDEF$ be a regular hexagon, and AC , BE intersect in G , show that EG is three times as long as BG .

9. Find a mean proportional between two given straight lines.

The straight line bisecting at right angles AC , one of the equal sides of an isosceles triangle, meets the base AB produced in D . Show that AC is a mean proportional between AB , AD .

10. Define *similar polygons*, and prove that they can be divided into the same number of similar triangles, having the same ratio to one another that the polygons have.

11. If the vertical angle of a triangle be bisected by a straight line which also cuts the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base, together with the square on the straight line which bisects the angle.

II. ARITHMETIC.

[Great importance is attached to accuracy.]

1. Multiply one million one thousand and one by one hundred thousand one hundred, and express the result in words.

2. Divide $99\cdot99$ by $\cdot0011$, and reduce $\frac{9 + \cdot9}{1\cdot9}$ to its simplest form.

3. Add together £175. 17s. 6d., five hundred guineas, eighty-seven half-crowns, and 1,143 four-penny pieces.

4. If a person buys 1 cwt. 2 qrs. 15 lbs. of goods for £2. 5s. $4\frac{1}{2}$ d. per cwt., what must he sell it for per cwt. in order to gain £6·88125 on the transaction?

5. Find the value of

$$\cdot54 \text{ of } 8s. \text{ } 3d. + \cdot027 \text{ of } £2. \text{ } 15s. + \cdot3125 \text{ of } £2. \text{ } 2s.$$

6. If the wages of 45 women amount to £207 in 48 days, how many men must work 16 days to receive £76. 13s. 4d., the daily wages of a man being double those of a woman?

7. Find the square root of $\cdot02010724$ and the cube root of 24137569 .

8. What sum will amount to £39. 12s. 6d. in 16 months at $4\frac{1}{2}$ per cent. per annum simple interest?

9. Find the discount on £663. 17s. 0d. due six months hence at 4 per cent.

10. What principal lent out at compound interest for 2 years at 5 per cent. will amount to £7425. 6s. 9d.?

11. A person sells his property for £27,118, the rental of which is £900; he invests £4818 in 3 per cents. at $100\frac{3}{8}$, £2450 in Greek securities at 50, 5 per cent., and £19,850 in New Zealand 4 per cent. at $99\frac{1}{4}$; find the increase of his income.

12. Find, by means of duodecimals, the number of feet of glass in a window which measures 7 feet $10\frac{3}{4}$ inches by 4 feet $7\frac{1}{2}$ inches.

13. Find the number of gallons in a cylindrical vessel whose diameter is 6 feet, and height 7 feet. Given that the area of a circle equals $\frac{2}{7}$ times the square of the radius, and that a gallon is equal to 277·2 cubic inches.

WOOLWICH ENTRANCE EXAMINATION.

14. An empty cistern has three pipes, *A*, *B* and *C*. *A* and *B* can fill it in three and four hours respectively, and *C* can empty it in one hour. If these pipes be opened in order at 1, 2 and 3 o'clock, find when the cistern will be empty.

15. A grocer mixes 17 lbs. of tea worth 4s. with 25 lbs. worth 4s. 8d., and sells the whole at 5s. 4d. per lb. ; what is his total gain and his profit per cent.?

16. A rectangular piece of ground 6 furlongs long and 4 furlongs broad is divided into four smaller rectangles by a belt 200 feet wide round the outside boundary, the inner part being divided by a road 60 feet wide in the direction of the length, and a cross-road 41 feet wide in the direction of the breadth. Find the area of the four rectangles in acres.

17. Reduce 2 lbs. 7 oz. 3 dwt. 15 grains troy to the decimal of a lb., and express 3 miles 7 furlongs 12 poles in French metres. A metre equals 39·371 inches.

18. Why must the decimal equivalent to $\frac{1}{5}$ recur? Find it, and the value of 1·683 lb. of gold, when it is worth £4·0099 per ounce.

19. In Réaumur's thermometer the freezing point is zero, and the boiling point 80; and in Fahrenheit's the former is 32 and the latter 212. What degree of Fahrenheit's corresponds to 9 of Réaumur?

20. What would be the logarithm of 180 if the base were 12?

$$\log 2 = \cdot 30103 \text{ and } \log 3 = \cdot 4771213.$$

21. Given the mantissae of the logarithms of the following numbers:—2289, 343, 1092, 854, 855677, to be ·3596458, ·5352941, ·0382226, ·9314579, ·9323102. Find the value of

$$\sqrt{2289 \times 343 \times 1092 \times 854}.$$

III. ALGEBRA.

[Great importance is attached to accuracy.]

1. Reduce each of the following expressions to its simplest form:—

(i.)
$$\frac{x}{x^2+y^2-xy} + \frac{1}{x+y} + \frac{2xy-x^2}{x^3+y^3};$$

(ii.)
$$\frac{a+b}{ab+c^2-ac-bc} + \frac{a+c}{ac+b^2-ab-bc} + \frac{b+c}{bc+a^2-ab-ac}$$

(observing that the denominators of these fractions may be written in factors).

(iii.)
$$\frac{x^2-1-\frac{6}{x^2}}{x^2-2x+3-\frac{4}{x}+\frac{2}{x^2}}.$$

2. Divide

$$x^4+y^4 \text{ by } x^2+y^2-\sqrt{2} \cdot xy; \text{ and } x^3+y^3+3xy-1 \text{ by } x+y-1.$$

3. If
- $z^2 = x^2 + y^2$
- find the simplest form of

$$\sqrt{(x+y+z)(x+y-z)(z+x-y)(y+z-x)}.$$

4. Solve the equations

(i.)
$$\sqrt{a-x} + \sqrt{b-x} + \sqrt{c-x} = \sqrt{a+b+c-x}.$$

(ii.)
$$\frac{x(x-1)}{(x+1)(x+2)} = \frac{(x+3)(x+4)}{(x+5)(x+6)}.$$

5. If
- α
- and
- β
- be the roots of the equation

$$ax^2+bx+c=0,$$

determine the equation whose roots are

$$\frac{\alpha}{\beta} \text{ and } \frac{\beta}{\alpha}.$$

6. The sum of two numbers is 12, and the sum of their fourth powers is 3026; find the numbers.

7. If
- $x' : y' : z' = xy : x^2 : yz$
- , prove that

$$x : y : z = x'y' : x'^2 : y'z'.$$

8. A rectangular enclosure is half an acre in area, and its perimeter is 201 yards; find the lengths of its sides.

9. The sum of 10 terms of an arithmetical series is 145, and the sum of its fourth and ninth terms is five times the third term; determine the series.

WOOLWICH ENTRANCE EXAMINATION.

10. If a series of numbers be in arithmetical progression, prove that their reciprocals are in harmonical progression.

The first two terms of a harmonic series are 4 and $\frac{8}{5}$, find its ninth term.

11. Find the coefficient of x^6 in the expression

$$\left[1 - \frac{x^2}{1 \cdot 2} + \frac{x^4}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{x^6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}\right]^2 + \left[x - \frac{x^3}{1 \cdot 2 \cdot 3} + \frac{x^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}\right]^2,$$

and the coefficient of x in the expansion of $(1+x)^{-\frac{1}{2}}$ by the Binomial theorem.

12. Find how many arrangements of letters can be formed with 8 unlike consonants and 4 unlike vowels, if each arrangement contains 3 consonants and 2 vowels; and prove the general theorem on which the result depends.

IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. State the greatest and least values of (1) the cosine (2) the secant of an angle; also to what angles these values respectively belong.

Show how to draw geometrically an angle whose secant = 3.

2. State and prove the formula expressing $\sin A + \sin B$ as a product; and show that the arithmetical mean of the sines of two angles is very nearly equal to the sine of the arithmetical mean of the angles, if the angles are nearly equal to each other.

3. Find an angle whose sine = $-\sin A$, and whose cosine = $-\cos A$.

4. Find the greatest possible value of $\sin x + \cos x$.

5. Prove that $\tan^{-1}t + \tan^{-1}\frac{2t}{1-t^2} = \tan^{-1}\frac{3t-t^3}{1-3t^2}$.

6. Prove the formula for a plane triangle

$$\cos \frac{1}{2} C = \sqrt{\frac{s(s-c)}{ab}}.$$

7. Given in a triangle $a = 250$, $b = 240$, $A = 72^\circ 4' 48''$, find the angles B , C . State whether they can have more than one value each.

$$\begin{array}{ll} \text{Given} & \log 2.5 = .3979400 & \log \sin 72^\circ 4' = 9.9783702 \\ & \log 2.4 = .3802112 & \log \sin 72^\circ 5' = 9.9784111 \\ & & \log \sin 65^\circ 59' = 9.9606739. \end{array}$$

8. Prove that in a triangle

$$\frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)} = \frac{a+b}{c}.$$

9. If in a triangle the bisector of the base, c , is perpendicular to the side b , prove that $2 \tan A + \tan C = 0$.

10. If p, q are the perpendiculars from A, B on any arbitrary line drawn through the vertex C of a triangle, prove that

$$a^2 p^2 + b^2 q^2 - 2abpq \cos C = a^2 b^2 \sin^2 C.$$

11. Explain what measurements have to be made at two stations A and B , in order to find the distance CD between two inaccessible objects, $ABCD$ being in one plane: and state clearly the steps of the calculation by which the distance is to be found therefrom.

12. If in a triangle $C = 60^\circ$, prove that

$$\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}.$$

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[In answering the questions on Geometry ordinary abbreviations may be employed, but the method must be geometrical. Full marks may be gained by answering correctly about three-fourths of this paper. Great importance will be attached to accuracy.]

1. If a straight line be at right angles to a plane, every plane which passes through it shall be at right angles to the plane.

If CD be at right angles to a plane which it meets in D , AB be any straight line in that plane, DM in the same plane perpendicular to AB , then any line drawn from M to the line CD is perpendicular to AB .

2. Prove that every solid angle is contained by plane angles which are together less than four right angles. Hence show that there are only five ways of forming a solid angle by the plane angles of the same regular rectilineal figure.

3. If a perpendicular be drawn from an angle of a regular tetrahedron upon the opposite triangular base, show that the foot of the perpendicular will divide either of the lines drawn from an angle of the base to the point of bisection of the opposite side in the ratio of 2 to 1.

4. If a right cone be cut by a plane, examine when the intersection of the cutting plane with the surface of the cone will be an ellipse, and find the positions of the foci of the ellipse for a given section.

5. Show when $ay^2 + 2cxy + bx^2 = 0$ represents two real, and different straight lines, and point out when it fails to do so. Show that the equations to the straight lines bisecting the angles between the lines represented by the above given equation may be expressed thus

$$\frac{x^2 - y^2}{b - a} = \frac{xy}{c}.$$

6. Show how to change the axes of rectangular coordinates to another rectangular system inclined at a given angle to the former and having the same origin. Show how the curve represented by $3x^2 + 2xy + 3y^2 = 1$ may be determined by turning the axes through 45° in the same plane.

7. If from a given point without a parabola tangents be drawn to the parabola, show that these tangents will subtend equal angles at the focus. Prove also that the directrix touches all circles described on focal chords as diameters.

8. In an ellipse prove that the normal at any point bisects the interior angle between the focal distances at that point, and that the focal distances make equal angles with the tangent.

If S and H be the foci of an ellipse, P any point in the curve, show that the ratio of $\tan \frac{PSH}{2} : \tan \frac{PHS}{2}$ is independent of the eccentricity of the ellipse, supposing the abscissa of P and major axis unchanged.

9. Define the polar of a point with respect to a circle, ellipse, or hyperbola; find its equation with respect to an ellipse when the point is without the curve, and show that the equation obtained represents the locus of the intersection of tangents drawn at the extremities of chords passing through a given point.

10. Assuming the ordinary rectangular equation to an hyperbola, find its equation referred to its asymptotes as axes. Prove that the portion of the tangent drawn at any point of an hyperbola intercepted by the asymptotes is bisected at the point of contact.

11. An angle of a cube is cut off by a plane which intersects the edges at distances (a) (b) (c) from the angle; if (α) be the angle of inclination of the cutting plane to the face of the cube, which is perpendicular to c , prove $\tan \alpha = \frac{c}{ab} \sqrt{a^2 + b^2}$. Hence prove also

$$(\cos \alpha)^2 + (\cos \beta)^2 + (\cos \gamma)^2 = 1,$$

where (β) and (γ) are the angles which the cutting plane makes with the other two plane faces of the cube respectively.

VI. PURE MATHEMATICS (2).

[Great importance will be attached to accuracy. Full marks may be obtained by answering three-fourths of this paper.]

1. Prove that, if x is any real quantity, $x + \frac{1}{x}$ is never less than 2, and find the least value of $x + \frac{1}{x-2}$.

2. If a , b , and c be positive quantities in harmonic progression, prove that $a + c$ is greater than $2b$, and that $a^n + c^n$ is greater than $2b^n$.

3. Expand $\frac{1}{(1-2x)(1-3x)}$ in ascending powers of x , and find the coefficient of x^n .

4. Find the sums of the series

$$(1) \quad 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots,$$

$$(2) \quad \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots,$$

each of n terms, and (2) to infinity.

5. Find all the positive integral solutions of the equation

$$9x + 11y = 455.$$

6. Reckoning compound interest, find an expression for the present value of a sum of M pounds due n years hence, at a given rate of interest.

Having given $\log 2 = .3010300$ and $\log 103 = 2.0128372$, find how many years will elapse before a sum of money doubles itself at 3 per cent. per annum compound interest.

7. If n_r represent the number of combinations of n things taken r together, prove that

$$(n+1)_r = n_r + n_{r-1}.$$

8. Find the number of different throws which can be made (1) with two ordinary dice, (2) with n ordinary dice.

9. Prove that for all values of n ,

$$(\cos \theta + \sqrt{-1} \sin \theta)^n = \cos n\theta + \sqrt{-1} \sin n\theta.$$

If $x_r = \cos \frac{\pi}{2^r} + \sqrt{-1} \sin \frac{\pi}{2^r}$, prove that, the product being continued to infinity,

$$x_1 \cdot x_2 \cdot x_3 \cdot x_4 \dots = \cos \pi.$$

10. Find the exponential expressions for the sine and cosine of an angle.

If $\log(1 + \sqrt{-1} \tan a) = A + B\sqrt{-1}$, prove that $A = \log \sec a$, and find B .

11. Prove that, in an equation with real coefficients, imaginary roots occur in pairs.

Having given that $2 + \sqrt{-3}$ is a root of the equation

$$x^4 - 21x^2 + 76x - 84 = 0,$$

find all its roots.

12. Explain Cardan's method of solving a cubic equation, and apply it to find, to two places of decimals, an approximate value of the real root of the equation

$$x^3 + 6x - 6 = 0.$$

VII. PURE MATHEMATICS (3).

[Full marks may be obtained for about three-fourths of this paper.]

1. Differentiate the expressions

$$\sin^{-1} \frac{x}{\sqrt{1+x^2}}, \log \sqrt{\frac{1-\cos x}{1+\cos x}}, x^x.$$

2. If $u = \sin^{-1} x$ prove that

$$(1-x^2) \frac{d^3 u}{dx^3} = 3x \frac{d^2 u}{dx^2} + \frac{du}{dx}.$$

3. Prove Maclaurin's theorem, and apply it to the expansions of $\sin x$, and of $\cos x$, in ascending powers of x .

4. Find the minimum value of $\frac{26+6x+9x^2}{1+3x}$, and the maximum or minimum values of $a \sin x + b \sin 2x$.

5. Find the equation of the tangent at any point on the curve

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}},$$

and prove that the portion of the tangent intercepted between the coordinate axes is of constant length.

6. If p be the perpendicular from the origin on the tangent to a curve at a point whose radius vector is r , prove that the radius of curvature at the point is $r \frac{dr}{dp}$.

Apply this to determine the length of the radius of curvature at any point on a parabola.

7. Trace the curve $r = a \cos 4\theta + b \sin 4\theta$, and find its entire area.

8. Determine the asymptotes to the curve

$$x^3 + 3x^2y + 2xy^2 + 3axy + 2ay^2 - 2a^3 = 0.$$

9. Integrate the expressions

$$\sin^3 x \cos^5 x \, dx, \quad x^2 \log x \, dx, \quad \frac{x^2 dx}{x^6 - 1}.$$

10. Find the values of *any two* of the definite integrals

$$\int_0^\infty \frac{dx}{1 + 2x \cos a + x^2}, \quad \int_0^1 \frac{dx}{\sqrt{x} + \sqrt{x+1}}, \quad \int_0^1 \sin^{-1} x \, dx.$$

11. Prove that the whole area of a cycloid is three times that of its generating circle.

12. Rectify *either* of the curves

$$(1) \quad ay^2 = x^3,$$

$$(2) \quad x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}.$$

VIII. STATICS.

[Great importance will be attached to accuracy.]

1. Two forces are given in magnitude, but may make any angle with each other. How should they be placed, so as to give (1) the greatest, (2) the least possible resultant? Prove your statements.

2. What is meant by a force resolved in a given direction? Show how a force is to be resolved so that its component along a given direction shall have a given value.

3. A force is given in magnitude and line of action. Give a geometrical construction for resolving it into two other forces which shall be equal to one another and shall pass respectively through two fixed points.

4. Prove that any two couples of equal moments and opposite senses balance each other. (It may be assumed that the same couple can be transferred in its own plane in any manner without changing its effect.)

5. Three parallel forces act on a horizontal bar. Each is = 1 lb.; the right-hand one acts vertically upwards, the two others vertically downwards at distances 2 ft. and 3 ft. respectively from the first; draw their resultant, and state exactly its magnitude and position.

6. Draw *any* system of pulleys by which a weight of 1 lb. can be made to support a weight of 3 lbs., neglecting friction and the weights of the pulleys. Show that, whatever may be your system, the smaller weight will descend through 3 ft. in raising the other through 1 ft.

7. A heavy circular disc is kept at rest on a rough inclined plane by a string parallel to the plane and touching the circle. Show that the disc will slip on the plane if the coefficient of friction is less than $\frac{1}{2} \tan i$, where i = slope of plane.

8. Two equal weights each = 112 lbs. are joined by a string which is laid over two pulleys A, B in the same horizontal line. If a small weight, say 1 lb., is attached to the string half way between A and B , find in inches the depth to which it descends below the level of AB : supposing $AB = 10$ ft.

What would happen if the weight were attached at any other point of the string?

9. If a portion m of any mass M is moved to any new position, show that the centre of gravity of the entire mass is thereby moved in a direction parallel to the displacement of the centre of gravity of m , and over a distance $= \frac{m}{M} D$, where D = above distance between the two positions of the centre of gravity of m .

A triangular piece of paper is folded across the line bisecting two sides, the vertex being thus brought to lie on the base. Find the centre of gravity of the paper in this position.

10. Three equal particles are placed anywhere on the three sides of a triangle. If they are moved along those sides, in the same sense, and over three spaces which are proportional respectively to the sides, show that the centre of gravity of the particles remains at rest.

11. Assuming the principle of virtual velocities, deduce the relation between the power and weight on the inclined plane; the power being either (1) parallel to the plane or (2) horizontal.

12. Explain what is meant by stable and unstable equilibrium, and give an instance of each.

IX. DYNAMICS.

[The measure of the acceleration of gravity may be taken to be 32 when a foot and a second are units of length and time. Great importance will be attached to accuracy.]

1. Explain how velocity is measured, and if 22 be the measure of a velocity when a foot and a second are units of length and time, find its measure when a mile and an hour are the units.

A man, 6 feet in height, walks with the velocity of 3 miles an hour, in a straight line along a road, on one side of which there is a lamp-post, the light of the lamp being 9 feet above the ground. Find the velocity of the end of his shadow on the ground.

2. A ship, which is sailing due north at the rate of 3 miles an hour, observes another ship exactly east of it, which is sailing due east at the rate of 4 miles an hour. Find the rate at which each ship is increasing its distance from the other, and determine graphically the direction of motion of each relative to the other.

3. Define acceleration, and explain how it is measured.

If a point, starting with no velocity, moves with a constant acceleration f in the direction of its motion, and passes over the space s in the time t , and if v is its velocity at the end of that time prove that

$$2s = ft^2, \text{ and } v^2 = 2fs.$$

If the point starts with the velocity u , and moving with the constant acceleration f , passes over the space s , where is the error in the statement that the final velocity is $u + \sqrt{2fs}$?

4. Enunciate and explain Newton's second law of motion.

A string passing over a smooth pulley supports two scale-pans at its ends, the weight of each scale-pan being equal to the weight of one ounce. If a two-ounce weight be placed in one scale-pan, and a four-ounce weight in the other, find the acceleration of the system, the tension of the string, and the pressures between the weights and the scale-pans.

5. Prove that the time of descent of a heavy particle down any chord of a vertical circle, starting from the highest point of the circle, is the same.

Find the line of quickest descent from a given point to a given circle in the same vertical plane.

6. Prove that the path of a projectile is a parabola, and find the greatest possible range on the horizontal plane through the point of projection for a given velocity of projection.

Show that, for any range short of the greatest, if the velocity of projection is given, there are two directions of projection which are equally inclined to the direction giving the greatest range.

7. If a point moves uniformly, with velocity v , in a circle of radius r , find the direction and the measure of its acceleration.

If a particle of mass m moves in the same circle with the same velocity, find the direction and magnitude of the resultant of the forces which are in action on the particle.

A heavy particle, which is suspended from a fixed point by a string one yard in length, is raised until the string (which is kept tight) is inclined 60° to the vertical, and is then projected horizontally, in the direction perpendicular to the vertical plane through the string; find the velocity of projection that the particle may move in a horizontal circle.

8. Find the velocity acquired by a heavy particle in sliding down a smooth curve.

A heavy particle is placed very near the highest point of a smooth sphere; find where it runs off the sphere, and prove that the latus rectum of the parabola, which it then describes, is eight twenty-sevenths of the diameter of the sphere.

9. Define the potential energy and the kinetic energy of a system, and enunciate the principle of energy.

A straight rod ACB , without weight, has two particles of equal weight fastened to it, one at the end B , and the other at the middle point C , and the rod can swing about the end A . If it be held horizontally, and then allowed to swing, prove that the greatest velocity acquired by the end B will be the same as the velocity acquired by a particle falling freely through a height equal to six-fifths of the length of the rod.

10. A cannon standing on a smooth horizontal plane is pointed horizontally and loaded with a ball, the mass of which is a given fraction of the mass of the gun and its carriage. Having given the velocity with which the ball leaves the gun when it is fired, find the velocity of recoil of the gun.

If the charge of powder be quadrupled, what will be the effect on the velocities?

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1886.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed ; but the method of proof must be geometrical. Great importance will be attached to accuracy.]

1. If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise their bases equal, the angle which is contained by the two sides of the one shall be equal to the angle, which is contained by the two sides, equal to them of the other.

2. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles ; and the three interior angles of every triangle are together equal to two right angles.

The angle contained between the two lines drawn from the vertex of a triangle, one to bisect the base and the other perpendicular to it, is equal to half the difference of the angles at the base.

3. The opposite sides and angles of a parallelogram are equal to one another, and the diameter bisects the parallelogram, that is, divides it into two equal parts.

4. In every triangle the square on the side subtending an acute angle is less than the squares on the sides containing that angle by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle, and the acute angle.

If D be the middle point of the base BC of a triangle ABC , E the foot of the perpendicular drawn to the base from A ; show that the rectangle DE , BC is half the difference of the squares on the sides of the triangle.

5. State and prove Euclid's proposition as to the relative lengths of straight lines drawn from any point within a circle to the circumference.

Two triangles ABC , DBC are on the same base BC and have their vertical angles, A and D , equal to one another. The angles at C being both acute, and the angle DCB being greater than the angle ACB , show that DB is greater than AB .

6. The angle at the centre of a circle is double of the angle at the circumference on the same base, that is, on the same arc.

On the base BC of a triangle ABC , which has an obtuse angle at B , describe another triangle equal to ABC and having its vertical angle double of the angle A .

Show that the construction is impossible unless $2C + A$ is less than a right angle.

7. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle.

8. Describe a square about a given circle.

The area of a regular octagon inscribed in a circle is equal to the rectangle contained by the sides of the inscribed and circumscribed squares.

9. Triangles and parallelograms of the same altitude are to one another as their bases.

10. In any right-angled triangle, any rectilineal figure described on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle.

$ABCD$ is a parallelogram whose side AB is divided in E so that AE is less than EB . CF (equal to AE) is drawn perpendicular to DC on the side remote from AB ; and FG (equal to EB) is drawn to meet DC produced in G . If GE and DA meet, when produced, in H ; show that the triangle HDG is equal to the parallelogram $ABCD$.

11. If from the vertical angle of a triangle a straight line be drawn perpendicular to the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the perpendicular and the diameter of the circle described about the triangle.

II. ARITHMETIC.

[N.B.—Great importance is attached to accuracy.]

1. The quotient in a division is 479, the dividend is 3476418, and the remainder is 794; what is the divisor?

2. Find the whole cost of 20 dozen boxes of fruit at 14s. $7\frac{1}{2}d.$ per box, 40 dozen at 13s. $9\frac{1}{2}d.$, and 60 dozen at 12s. $8d.$

3. Prove from the definition of a fraction that $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$. State in words the rule that you deduce for the addition or subtraction of fractions with unlike denominators.

4. Express in tons, cwts., etc., the value of

$$116\frac{1}{8} \text{ cwts.} + 115\frac{1}{4} \text{ qrs.} + 152\frac{1}{8} \text{ lbs.} + 111\frac{1}{2} \text{ oz.}$$

5. From $5\frac{1}{2}$ of '9625 subtract $3\frac{3}{8}$ of 1'097; and express as a decimal in its simplest form the value of $\frac{.1 + .011 + .001}{.41} \div .037$.

6. Find by practice the rent of 47 acres 2 roods 36 poles $2\frac{3}{4}$ yards at £20. 3s. 4d. per acre.

7. If 6 women and 3 boys weed $\frac{7}{12}$ of a field in 4 days, how long will it take 4 women and 1 boy to weed the rest of it, the work of 4 women being equal to that of 5 boys?

8. Find the square root of $12787\frac{1}{4}$; and the cube root of 112678587.

9. Find the simple interest on £1024. 7s. 6d. for 4 years 9 months, at 4 per cent. per annum.

10. Find the present value of a bill for £174. 1s. $10\frac{1}{2}d.$ due in 128 days, interest being at the rate of 5 per cent. per annum.

11. How much stock at $97\frac{1}{2}$ must I sell out in order to realize £1300? And if, by investing this £1300 in $4\frac{1}{2}$ per cent. stock at 102, I increase my income by £8. 6s. 8d. per annum, what rate of interest was I receiving on the former stock?

12. Find by duodecimals the cubical content of a cistern whose internal dimensions are 6 ft. $6\frac{1}{2}$ in., 5 ft. $3\frac{1}{4}$ in., and 4 ft. 8 in. respectively; and reduce the answer to cubic yards, feet, and inches.

WOOLWICH ENTRANCE EXAMINATION.

13. If a man buys eggs at 10 for a shilling, and sells them at 8 for a shilling, what rate per cent. profit does he make on his outlay?

14. If 4 men can do a piece of work in 7 days, which 5 women can do in 8 days, or 7 boys can do in 10 days, how long will it take 3 men, 2 women, and 3 boys to do the same if they work together?

15. Determine the weight in pounds of a box, with a lid, made of wood $\frac{3}{4}$ of an inch thick and measuring externally, 4 ft. by 3 ft. by 3 ft., the weight of a cubic foot of the wood being 38·4 lbs.

16. What length of carpet, 30 inches wide, will be required for a room 17 ft. 4 in. long and 13 ft. 9 in. wide? And, if the cost of the carpet is £5. 19s. 2d., what is the price of it per yard?

17. If the annual increase in the population of a state is 25 per thousand, and the present number of inhabitants is 2,624,000, what will the population be in 3 years' time? And what was it a year ago?

18. A man having 3 sons, left £9656 to be divided among them in proportion to their ages at the time of his death; when he died, their ages were 25, 22, and 21 years respectively; what was the share of each? And what difference would it have made to each of the sons if the father had lived a year longer?

19. Find to the nearest farthing the price per lb. in English money of tea which costs in France 3·50 francs per kilogramme; £1 being equivalent to 25 francs, and 1 kilogramme to 2·204 lbs.

20. Reduce $\frac{5}{11}$ of a pole (linear) to the decimal of $\frac{1}{88}$ of a furlong; and find the value of 0·01481 of a ton, at 4s. 6d. per lb.

21. (1) Find from the tables supplied log 637 and log 13; from these two logarithms determine log 7.

(2) Find from the tables supplied log 12 and log 18; from these two logarithms determine log 2 and log 3.

22. Find by aid of the tables the value of

$$\frac{87\cdot327 \times 784\cdot55 \times \cdot020868}{\cdot61659 \times 58\cdot844}.$$

III. ALGEBRA.

[N.B.—Great importance is attached to accuracy.]

1. Divide
- $1 + 4a^4b^4$
- by
- $2a^2b^2 - 2ab + 1$
- : and

$$x^4 + x^{-4} + 4(x^3 + x^{-2}) + 6 \text{ by } x^2 + x^{-2} + 2.$$

2. Express in their simplest forms

$$(i.) \left(1 - \frac{2xy}{x^2 + y^2}\right) \div \left(\frac{x^3 - y^3}{x - y} - 3xy\right).$$

$$(ii.) (x - y + z)(x + y - z) - (x + y + z)(x - y - z) - 4yz.$$

$$(iii.) \frac{x^3 - 2ax^2 - 5a^2x - 12a^3}{x^3 - 7ax^2 + 13a^2x - 4a^3}.$$

3. If
- $a^2 = bc$
- , prove that
- $b(c^2 - ab)^2 + c(b^2 - ca)^2 + 2a(b^2 - ca)(c^2 - ab) = 0$
- .

4. Solve the equations

$$(i.) \sqrt{1 + \frac{bx}{a^2}} + \sqrt{1 - \frac{bx}{a^2}} = 1\frac{3}{5}.$$

$$(ii.) \begin{cases} x^2 + y^2 = xy + 7 \\ x^2 - y^2 = xy - 1 \end{cases}.$$

5. 1120 square feet of paper will just cover the four walls of a room which is 8 feet longer than it is wide. If the room were 4 feet higher the same quantity of paper would just cover the two smaller and one of the larger walls. What are the dimensions of the room?

6. If α and β are the roots of the equation $x^2 + px + q = 0$, and $\alpha - \gamma$, $\beta - \gamma$, those of $x^2 + Px + Q = 0$, show that $4q - p^2 = 4Q - P^2$.

7. Two cyclists travel, one from A to B , the other from B to A , by the same road, and at uniform speeds. They start at the same moment. One reaches B $2\frac{1}{2}$ hours, the other reaches A 3 hours 36 minutes after they meet. How long was each on the journey?

8. Explain and justify the process ordinarily used for finding the square root of an algebraic expression.

Show how the method is applied to numerical examples.

9. Show how to find the sum of n terms of a given Geometrical Progression. If x, y, z be in G.P., prove that

$$x^2y^2z^2(x^{-3} + y^{-3} + z^{-3}) = x^3 + y^3 + z^3.$$

10. The arithmetical mean between two numbers is to the geometric mean as 5 to 4, and the difference of their geometrical and harmonical means is $3\frac{1}{2}$; find the numbers.

11. If m and n are positive integers, show that the coefficient of x^m in the expansion of $(1+x)^{m+n}$ is equal to that of x^n .

Find the coefficients of x^3 and x^5 in the expansion of $(1-2x-3x^2)^{-2}$.

12. Assuming the formula for the number of permutations of n different things taken all together, show how the number may be found when the things are not all different.

Hence find how many different numbers may be composed with the figures in 111223, each consisting of 6 figures.

IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Explain what is meant by the *circular measure* of an angle, and show that the circular measure of an acute angle is intermediate in value between the sine and the tangent of the angle.

The perimeter of a certain sector of a circle is equal to the length of the arc of a semicircle having the same radius. Express the angle of the sector in degrees, minutes, and seconds.

2. Prove that the secant of any angle will be either greater than $+1$ or less than -1 .

Show that $\sec A = \pm \sqrt{1 + \tan^2 A}$, and explain the appearance of the double sign.

3. Express $\sin(A-B)$ and $\cos(A-B)$ in terms of the sines and cosines of A and B .

Find the values of $\sin 15^\circ$, $\sin 105^\circ$, $\cos 165^\circ$, and $\tan 195^\circ$.

4. Obtain a formula including all angles which have a given tangent.

If $\tan A + \tan 2A = \tan 3A$, prove that A must be a multiple of 60° or 90° .

5. A ring, 10 inches in diameter, is suspended from a point 1 foot above its centre by six equal strings attached to its circumference at equal intervals. Find the cosine of the angle between two consecutive strings.

6. Prove that

$$(1) \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}.$$

$$(2) \cos^2 A - \cos A \cos (60^\circ + A) + \sin^2 (30^\circ - A) = \frac{3}{4}.$$

7. State and prove the rule for finding, by inspection, the characteristic of the logarithm of a fraction.

Find from the tables supplied an approximate value of the seventh root of .000026751.

8. Express the cosine of any angle of a triangle in terms of the three sides.

Prove that

$$\sin (B - C) : \sin (B + C) :: b^2 - c^2 : a^2.$$

9. The sides of a triangle are 17, 20, and 27. Find from the tables supplied all the angles.

10. A man travelling due west along a straight road observes that when he is due south of a certain windmill the straight line drawn to a distant tower makes an angle of 30° with the direction of the road. A mile further on the bearings of the windmill and tower are N.E. and N.W. respectively. Find the distances of the tower from the windmill, and from the nearest point of the road.

11. Find the area of a regular polygon of n sides inscribed in a circle of given radius.

If a regular pentagon and a regular decagon have the same perimeter, prove that their areas are as $2 : \sqrt{5}$.

FURTHER EXAMINATION.

V. PURE MATHEMATICS (I).

[In answering the questions on Geometry ordinary abbreviations may be employed, but the method of proof must be geometrical. Great importance will be attached to accuracy.]

1. If three straight lines meet all at one point, and a straight line stand at right angles to each of them at that point, the three straight lines shall be in one and the same plane.

2. If a plane A be parallel to a plane B , and a plane C to a plane D , show that the straight line which is the intersection of A and C is parallel to the straight line which is the intersection of B and D .

3. Prove that the line of intersection of two spheres is a circle. If any other sphere includes this circle upon its surface, find the locus of its centre.

4. Through a point P on a parabola the tangent, normal, and ordinate are drawn. Show that the portion of the axis intercepted between the tangent and normal is twice the portion of the axis intercepted between the ordinate and the directrix.

5. A parabola is drawn to touch two sides of a triangle at the angular points upon the third side. Compare (with proof) the areas of the portions into which the triangle is divided.

6. Being given four tangents to a parabola in position, determine its focus and directrix by geometrical construction.

7. Define an *asymptote*. Prove that the area of all triangles formed by the asymptotes and a tangent to one branch of a given hyperbola is the same.

From the points B, C , in which a tangent to an hyperbola meets the asymptotes, CH, BG , are drawn parallel to one another to meet the asymptotes in H and G . If AFD is a straight line parallel to CH and meeting the asymptote HB in A and the asymptote CG in D and the tangent in F , show that it also will be a tangent to the hyperbola when the triangle AFG is equal to the triangle HFD .

8. Find the condition that the straight lines $Ax + By = 1$ and $ax + by = 1$ may be (1) perpendicular; (2) parallel. In the latter case find the distance between the straight lines.

9. The four angular points of a quadrilateral referred to oblique coordinates are (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , (x_4, y_4) . Find the conditions that it may be (1) a rhombus, (2) a square.

10. If $\alpha = 0$, $\beta = 0$, $\gamma = 0$ represent, in abridged notation, the three sides of a triangle, show that any right line can be represented by

$$l\alpha + m\beta + n\gamma = 0.$$

If a , b , c , are the sides of the triangle, and $l : m : n :: a : b : c$, comment on the above equation.

11. Find the equations to the tangent and normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at a point of which the eccentric angle is θ . What value must be assigned to the coefficient m in $x^2 - y^2 = mxy$, in order that this equation may represent straight lines through the origin parallel to the normal and tangent at the point of which the eccentric angle is θ ?

12. Examine the nature of the following curves, and trace them :

(i.) $(x+a)^2 = c^2 + (y+b)^2$.

(ii.) $(x+y-7)(x-y-3) = 21(x-1)^2$.

(iii.) $x^3 - y^3 = (y-a)(x^2 - y^2)$.

VI. PURE MATHEMATICS (2).

1. If x and y are real quantities, prove that

$$(xy - 1)^2 > (x^2 - 1)(y^2 - 1).$$

2. If x and y are both positive, and if

$$x < \frac{1-y}{1+y}, \text{ prove } y < \frac{1-x}{1+x}.$$

3. If a number N is divisible by 11, prove that

$$a - b + c - d + \text{etc.}$$

(if not 0) is divisible by 11; a , b , c , d ... being the digits composing N , beginning from the right.

4. In how many different orders may six people be formed into a ring?

A party of n ladies and n gentlemen sit round a table. How many different arrangements are possible if each lady is to sit between two gentlemen?

5. Show that

$$\phi(x) = a + \frac{\log(\log x)}{\log r}$$

is a function of x which satisfies the condition

$$\phi(x^r) \equiv 1 + \phi(x).$$

6. Find a quadratic whose roots are the squares of the roots of

$$x^2 - 2x \cos \theta + 1 = 0.$$

7. Find in any manner the three roots of the cubic

$$4x^3 - 27(x-1)^2 = 0.$$

8. Expand $(1+x+x^2+x^3)^{-1}$ according to powers of x .

9. What is the trigonometrical value of $e^{\theta} \sqrt{-1}$?

Show that, if $\theta < 90^\circ$,

$$\sqrt{-1} \cos^{-1}(\cos \theta + \sin \theta) \text{ is real.}$$

10. If a point Z be taken in the base AB of a triangle ABC , such that

$$AZ : ZB :: AC^2 : BC^2,$$

prove that ZC divides the angle C into two parts, which are equal to those into which C is divided by the bisector of the base.

11. To a person going northward, when at A , two objects B , C , appear in a straight line, pointing N.W. After advancing a distance a the object C still seems almost due N.W. of him, while the angle which B and C subtend at his eye is found to be θ . Find approximately the distance AB .

12. In the equation

$$x^n + ax^{n-1} + bx^{n-2} + \dots = 0,$$

the coefficients a , b , c are integers. Prove that it cannot have a fractional root.

VII. PURE MATHEMATICS. (3).

[Full marks may be obtained for about three-quarters of this paper.]

1. Explain briefly the doctrine of limits on which the differential calculus depends.

Find the differential coefficient of $\tan x$, assuming, if it be needed, that the limit, when $x = 0$, of $\frac{\tan x}{x}$ is 1.

2. What is understood by different orders of infinitesimals? Show that if one side of a right-angled triangle be regarded as an infinitesimal of the first order, the difference between the hypotenuse and the remaining side is an infinitesimal of the second order.

Two houses A and B lie north and south of one another at a distance of 100 yards apart. A third house C lies 2 miles from B in a south-west direction. A postman calling at C can either proceed directly to A and thence to B , or he can take the opposite course. How much shorter (approximately) is one way than the other?

3. Obtain an expansion for $\sin^{-1}x$ in terms of x as far as the term involving x^5 , and write down the general term of the series. From three terms of this series, obtain an approximate value of π .

4. If $y=f(x)$, show generally that the values of x which are derived from the equation $\frac{dy}{dx}=0$ give the maxima and minima values of (y) , and show how to distinguish the maxima from the minima.

AB, AC are two straight lines meeting in A ; M is a point between them: draw through M a straight line PMQ meeting AB and AC in P and Q respectively, so that the area of the triangle PAQ may be a minimum.

5. In a plane curve determine the lengths of the portions of the tangent and normal respectively intercepted between the point of contact and the axis of (x) .

In an ellipse find the points in the curve at which these lengths of the tangent and normal are equal.

6. Find an expression for the radius of curvature of a plane curve in rectangular coordinates; determine it at any point of the catenary whose equation is $y = \frac{c}{2} \left(e^{\frac{x}{c}} + e^{-\frac{x}{c}} \right)$, and write down its value at the lowest point of the curve.

7. If (p) be the perpendicular from the pole of a curve referred to polar coordinates, prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta} \right)^2$.

In a parabola, whose polar equation is $r = \frac{2m}{1 + \cos \theta}$, obtain, from the above, the equation between (p) and (r) .

8. Trace the curve $y^2 = x^2 \cdot \frac{x-a}{x-2a}$; draw its asymptotes, and determine if the ordinates have maxima or minima values.

9. There are certain elementary forms of differentials whose integrals are known from inspection, give the integrals of any four such differentials.
Integrate

$$(1) \frac{x dx}{x^2 - 7x + 12}; \quad (2) \sqrt{a^2 - x^2} \cdot dx.$$

10. Prove that

$$\int_0^{\frac{\pi}{2}} (\sin x)^6 dx = \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{\pi}{2}.$$

11. Find the differential expression for determining the area of a plane curve in polar coordinates.

Find the area of the loop of the curve whose equation is $r^2 = a^2 \cos 2\theta$.

12. Find the content of a given sphere.

Prove that the content of a sphere is to the content of the greatest cone that can be inscribed in it as 27 to 8.

VIII. STATICS.

1. How are forces measured in Statics? Can three forces, proportional to 9, 5, 3 respectively, acting in one plane on a particle, be so arranged as to be in equilibrium? State and prove any proposition that involves the answer to the question.

2. Two equal forces inclined to each other at a given angle act on a fixed point; find the pressure on the point.

Two equal weights (W) are attached to the extremities of a thin string which passes over three tacks in a wall arranged in the form of an isosceles triangle with the base horizontal, the vertical angle at the upper tack being 120° ; find the pressure on each tack.

3. If a straight uniform rod is suspended by a thin string fastened to its middle point, and be kept in equilibrium by two weights on the opposite sides of the middle point, find the tension of the string by which the rod is suspended, and show how the weights are related to each other.

A straight lever two feet long is movable about a hinge at one end, and is kept in a horizontal position by a thin vertical string attached to the lever at a distance of 8 inches from the hinge, and fastened to a fixed point above the lever; if the string can just support a weight of 9 ounces without breaking, find the greatest weight that may be suspended from the other end of the lever.

4. Show that any system of forces acting on a rigid body in one plane may be reduced to a single force and a single couple. A rod is placed in *any* given position with one end on a smooth floor, and the other end against a smooth wall. Find a single force and a single couple which together will keep it at rest in that position.

5. If a right cone be cut by a plane bisecting its axis, find the distance of the vertex of the cone from the centre of gravity of the frustum thus cut off.

6. In that system of three pulleys (usually called the third system) where each string is attached to the weight and the weights of the pulleys are all equal, find the relation of the power to the weight, when equilibrium is established. If each pulley weighs 2 ozs. what weight would be supported by the pulleys alone?

If the weight supported be 25 lbs. and the power 3 lbs., find what must be the weight of each pulley.

7. Find the force acting up and parallel to a given rough inclined plane that is just able to move a weight up the plane.

Two equal weights are attached to a string that is laid over the top of two inclined planes having the same altitude and placed back to back, the angles of inclination of the planes being 30° and 60° respectively, show that the weights will be on the point of moving if the coefficient of friction between each plane and weight be $\frac{1}{2+\sqrt{3}}$.

8. State the principle of virtual velocities. Assuming the principle true, deduce from it the condition of equilibrium on a bent lever of unequal arms when acted on by weights suspended at the extremities of the arms, and show that for an indefinitely small displacement the centre of gravity of the weights will neither ascend nor descend.

9. A lamina in the form of an isosceles triangle, whose vertical angle is α , is poised upon a sphere, radius r , so that its plane is vertical and one of its equal sides (a) is upon the surface of the sphere; show that the equilibrium will be stable in the plane of the triangle if $\sin \alpha$ be less than $\frac{3r}{a}$.

10. Define the unit of work, and explain how it varies with the units of length, mass, and time.

A chain weighing 8 lbs. per foot is wound up from a shaft by the expenditure of four million units of work; find the length of the chain.

11. A cylindrical shaft has to be sunk to a depth of 100 fathoms through chalk whose specific gravity is 2.3; the diameter of the shaft being 10 feet. What horse-power is required to lift out the material in 12 working days, of 8 hours each? [The weight of a cubic foot of water is 62.5 pounds, and one horse-power is 33,000 foot-pounds a minute.]

IX. DYNAMICS.

[The measure of the acceleration of gravity may be taken to be 32 when a foot and a second are units of length and time.]

[Great importance will be attached to accuracy.]

1. State the theorem known as the *parallelogram of velocities*.

A stream runs with a velocity of $1\frac{1}{2}$ miles an hour; find in what direction a swimmer, whose velocity is $2\frac{1}{2}$ miles an hour, should start, in order to cross the stream perpendicularly.

What direction should he take to cross in the shortest time?

2. A person walking along a road at a rate v , sees a tower a mile distant from his eye; the nearest distance of the tower from the road is half a mile. Find the rate at which he is approaching the tower. Does this rate alter as he advances?

3. A sphere at rest is struck directly by another in motion. Describe in a few words the essential points of the phenomenon of impact, (1) when the spheres have no elasticity, (2) when they are perfectly elastic.

4. A particle is shot vertically upwards with velocity u ; find the time of ascent, and show that it is equal to the time of descent.

5. If a number of particles are let fall at the same instant down a number of smooth inclined planes having a common vertex, prove that at any moment they all lie on one circle.

Prove also that if all the planes are equally rough, the particles will be on one circle.

6. A ball is discharged with the initial velocity of 1100 feet; find in miles the greatest range it can give on a horizontal plane.

Could this range be reached in practice?

7. A projectile is thrown with given velocity and elevation, give the horizontal and vertical components of its velocity after a given time.

If after a given time a second projectile is thrown from the same point, in the same trajectory, describe what appearance its motion would present, as seen from the first one.

8. One end of a string is fixed ; it then passes under a movable pulley to which a weight, W , is attached. The string then passes over a fixed pulley, and a (smaller) weight, P , is attached to its other end, all three sections of the string being vertical. Show that, neglecting the masses of the pulleys, the acceleration with which W descends is

$$f = \frac{W - 2P}{W + 4P}g.$$

Verify this result when P is small compared to W ; and when W is small compared to P .

9. A bead is strung on a thread whose two extremities are fixed, and on which it can move without friction ; show that if the bead be started so as to move in the elliptic arc to which it is constrained, its velocity will be uniform (gravity not acting).

Show also that the tension of the thread, for different positions of the bead, varies inversely as the product of the two focal radii vectores.

10. Define the *work* done by a force when it continues to act while its point of application moves over a given space in the direction of the force. If that point moves at an angle with the force, what is the work done ?

A mass m is moving with velocity u . A constant force acts on it in the direction of the motion until its velocity is increased to v . Prove that the work done by the force is

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2.$$

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1887.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed, but the method of proof must be geometrical. Great importance will be attached to accuracy.]

1. Upon the same base and on the same side of it there cannot be two triangles which have their sides that are terminated at each extremity of the base equal to one another.

2. If a side of a triangle is produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are equal to two right angles.

In the triangle ABC , AD and AE are drawn from the vertex A to the base BC , making the angle BAD equal to C , and the angle CAE equal to B : prove that the perpendicular from A upon BC bisects DE .

3. Describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given rectilineal angle.

WOOLWICH ENTRANCE EXAMINATION.

4. If a straight line is divided into two equal parts, and also into two unequal parts, the rectangle contained by the unequal parts, together with the square of the part between the points of section, is equal to the square of half the line.

Show that the greatest right-angled triangle, which has the sum of the sides containing the right angle equal to a given straight line, is isosceles.

5. If a straight line touches a circle, the straight line which joins the centre to the point of contact is perpendicular to the touching line.

$ABCD$ is a straight line; circles are described on AB and CD as diameters; and a common tangent to the circles is drawn, meeting them in E and F . Prove that the triangles AEB and CFD are equiangular to one another.

6. The angle in a semi-circle is a right angle, and the angle in a segment greater than a semi-circle is less than a right angle, and the angle in a segment less than a semi-circle is greater than a right angle.

7. If two straight lines cut one another within a circle, the rectangle contained by the segments of one of them is equal to the rectangle contained by the segments of the other. Prove this in the one case only in which one of the straight lines passes through the centre and cuts the other, which does not pass through the centre, not at right angles.

AB , AC are equal chords of a circle; and AED is any chord through A , cutting BC in E ; prove that the rectangle $AD \cdot AE$ is equal to the square of AB .

8. Inscribe an equilateral and equiangular pentagon in a given circle.

The five lines, other than the sides of the figure, which can be drawn connecting the angular points of an equilateral and equiangular pentagon include between them a pentagon which is itself equilateral and equiangular.

9. If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides, or these produced, proportionally; and, conversely, if two sides, or these produced, be cut proportionally, the straight line which joins the points of section shall be parallel to the remaining side of the triangle.

Straight lines AOB , COD , intersect in O , and $AO : OB :: CO : OD$; if P , Q , are the middle points of AB , CD , prove that PQ is parallel to AC and BD .

10. Equal triangles, which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional; and, conversely, triangles which have one angle of the one equal to one angle of the other, and their sides about the equal angles reciprocally proportional, are equal to one another.

11. From the vertex A of a triangle ABC , AD and AE are drawn to the base, making the angle BAD equal to the angle CAE ; prove that the rectangle $BD \cdot BE$: the rectangle $CE \cdot CD :: AB^2 : AC^2$.

12. Describe a rectilinear figure which shall be equal to one and similar to another given rectilinear figure.

II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—*Great importance is attached to accuracy.*]

1. What is the smallest number which when subtracted from 99099, will make it exactly divisible by 909?

2. What will $37\frac{1}{2}$ lbs. of nutmegs cost at 1s. $6\frac{1}{2}d.$ per oz.?

3. What is meant by (a) the numerator, (b) the denominator of a fraction? Prove that $\frac{3}{4} = \frac{6}{8}$, and add together $\frac{3}{10} + \frac{7}{12} + \frac{6}{24}$.

4. If 13 yards of cloth cost £2. 0s. 1d., what must be given for 75 yards? Do this by Rule of Three, and explain the principle on which your statement is made.

5. If £150 gain £3. 7s. 6d. in 9 months, what sum will gain £3 in 12 months?

6. Find by Practice the price of 22 cwts. 3 qrs. 21 lbs. at £2. 9s. 6d. per cwt.

7. Multiply '0047 by '00035, and prove the correctness of your result; and divide '00918 by '018.

8. Find the value of '05854 of a guinea, and reduce 3 weeks 4 days 5 hours 6 minutes 6 seconds to the decimal of a month.

WOOLWICH ENTRANCE EXAMINATION.

9. Extract the square root of 63409369, and the cube root of $66430\cdot125$.
10. The interest of £25 for $3\frac{1}{2}$ years at simple interest was found to be £3. 18s. 9d.; what was the rate per cent.?
11. What is the difference between the compound interest on £100 for 2 years, according as the interest is paid yearly or half-yearly, 4 per cent.?
12. Define Present Worth and Discount: and find the proportion between the interest of £100 for 4 years, at $6\frac{1}{2}$ per cent., and the discount of the same sum payable at the end of 4 years, at the same rate of interest.
13. A person sells an estate worth £1200 per annum for 25 years' purchase, and after deducting $1\frac{1}{2}$ per cent. for expenses invests the remainder in North-Eastern Consols at 155; allowing 2s. 6d. per cent. for brokerage, find the amount of stock he will receive. If he gets 7 per cent. on his investment what will be the difference of his income, supposing the management of his estate to have cost him 10 per cent. of the rental?
14. What are meant by duodecimal fractions? By means of duodecimals, find the area of a rectangle 2 ft. 3 in. by 5 ft. 7 in., and express your answer in square feet and inches. And find the cost of glazing a window containing 60 panes, each of which is 1 ft. 2' 3" by 11' 5", at 3s. 8d. per square foot.
15. A tradesman forms a mixture of tea by adding 1 lb. at 3s. and $\frac{1}{2}$ lb. at 4s. 6d. to every 2 lbs. at 2s.; what must he sell it at per lb. in order to gain 10 per cent.?
16. A rectangular wooden box *exactly* contains 6 iron balls, each 10 inches in diameter, packed in sawdust: the wood is 1 inch thick. Find the weight of the whole; given that a cubic foot of wood or sawdust weighs 38'4 lbs.; a cubic inch of iron weighs 4'56 oz.; and that the volume of a sphere is $\frac{1}{6}\pi$ times the cube of its diameter.
17. What fractions produce circulating decimals? What kind of a decimal is $\sqrt{2}$ equivalent to? Convert $\cdot 05\dot{4}$ into a fraction, and explain your method of doing so. Multiply together $\cdot 4$, $\cdot 4$, $\cdot 0004$, and find the result to the end of the first recurring period.
18. Two clocks are together at 12 o'clock: one loses 7" and the other gains 8" in 12 hours: when will one be half-an-hour before the other, and what o'clock will it then show?
19. Find the value of $16\sqrt{3} + 10\sqrt[3]{4} - 4\sqrt{12} - 3\sqrt[3]{108}$ to 2 places of decimals.

20. Given $\log 2 = \cdot 3010300$, $\log 3 = \cdot 4771213$, $\log 1\cdot38 = \cdot 1398791$, find $\log \frac{8}{9}$, $\log 2\cdot16$, $\log \cdot 006$, and the value of $\sqrt{\frac{3^3 \sqrt{138}}{\sqrt[5]{\cdot 01}}}$. $\log 6\cdot240325 = \cdot 7952071$. Between what powers of 10 does 2^{64} lie?

21. If the side of a cube be 8, find the side of another cube exactly double the volume of the former. Given $\log 11 = 1\cdot04139$.

III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—Great importance is attached to accuracy.]

1. Multiply

$$ax^2 + ax + \frac{1}{a} \text{ by } \frac{x^2}{a} - ax + a,$$

and divide

$$x^4 + 5ax^3 - (n^2 + n - 7)a^2x^2 - 5na^3x - 6a^4 \text{ by } x^2 - (n - 2)ax - 2a^2.$$

2. Find the Highest Common Factor of

$$x^5 - x^3 + 4x^2 - 3x + 2 \text{ and } 5x^4 - 3x^2 + 8x - 3.$$

3. Simplify

$$(1) \frac{1}{axy} - \frac{1}{a(x-a)(y-a)} + \frac{1}{y(x-y)(y-a)}.$$

$$(2) \left\{ 1 + \frac{2b^2}{a(a-3b)} \right\} \left\{ 1 + \frac{b}{2b-a} \right\} - \left(\frac{a^2}{b^2} + \frac{b}{a} \right) \left(\frac{a^2 - ab}{a^2 - ab + b^2} - 1 \right).$$

4. Express $\{17 - 3\sqrt{21}\}^{\frac{1}{3}}$ as the difference of two surds, and find the value of

$$\frac{\sqrt{x} + \sqrt{x - \frac{1}{x}}}{\sqrt{x} - \sqrt{x - \frac{1}{x}}} \text{ when } 2x = \sqrt{a} + \frac{1}{\sqrt{a}}.$$

WOOLWICH ENTRANCE EXAMINATION.

5. Find the condition that the roots of the equation $ax^2 - bx + c = 0$ may be real and unequal.

Also show that, if one root is the square of the other,

$$b = \sqrt[3]{a^2c} + \sqrt[3]{ac^2}.$$

6. Solve the equations

$$(1) \quad \begin{cases} \frac{x+4}{7} - \frac{x-y-1}{4} = 2x-4. \\ 2y-4 - \frac{3x-2y}{3} = 3x. \end{cases}$$

$$(2) \quad \frac{15}{x+2} - \frac{8}{x-1} = \frac{3}{x-4}.$$

$$(3) \quad ax + b\sqrt{x^2 + bx + ab} = a^2.$$

7. If

$$a : b :: b : c :: c : d,$$

prove that

$$a^2 : d^2 :: a^3 : c^3.$$

8. Show how to insert any number of arithmetic means between two given quantities.

$2n$ arithmetic means are inserted between a and b : find the middle pair of means.

9. Sum to $2n$ terms each of the series

$$1 - 3 + 9 - 27 + \text{etc.},$$

$$1 - 3 + 5 - 7 + \text{etc.},$$

and write down the last term of each series.

10. How many different selections of five young trees can be made from a plantation containing 45?

Also, if five trees be selected from each of six different plantations, in how many ways can they be planted in six rows, each consisting of trees from the same plantation?

11. State the Binomial Theorem, and, assuming it to be true for any positive index, show that it will also be true for any negative index.

Find the coefficients of x^6 in $(1+x)^{-3}$ and $(1+x+x^2)^{-3}$.

12. A man's net income is 10 per cent. less than his gross income, which is derived partly from land and partly from personal property. He pays income tax at 8*d.* in the £ on the whole, and also local rates on the income from land. If his whole income had been derived from land, his net income would have been one-ninth less. What are the local rates per £, and in what proportion is his gross income derived from the two kinds of property?

IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Give a definition of the tangent of an angle which will apply to angles of all magnitudes; and trace the changes in the value of the tangent as the angle changes from 0° to 360° .

If $\tan A = \frac{3}{4}$, find all the other trigonometrical functions of A .

2. Prove by means of a figure that

$$\cos(A - B) = \cos A \cos B + \sin A \sin B;$$

the angle A being between 90° and 135° , and B between 45° and 90° .

3. If $\tan A = \frac{\sqrt{3}}{4 - \sqrt{3}}$, and $\tan B = \frac{\sqrt{3}}{4 + \sqrt{3}}$;

prove that $\tan(A - B) = .375$.

4. Find the values of $\sin 60^\circ$; $\sin 165^\circ$; $\sin 18^\circ$.

5. Prove that

$$\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}.$$

6. Prove that in *any* triangle

$$(i.) \quad a \sin B = b \sin A.$$

$$(ii.) \quad a = c \cos B + b \cos C.$$

7. Assuming $\cos A = \frac{b^2 + c^2 - a^2}{2bc},$

prove that $\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$

If $a = 40$, $b = 51$, $c = 43$; find the value of A , having given

$$\log 128 = 2.107210.$$

$$\log 603 = 2.780317.$$

$$\log \tan 24^\circ 44' 16'' = 9.6634465.$$

8. Given $\log 2 = .30103$, and $\log 3 = .47712$; find $\log \sin 60^\circ$ and $\log \tan 30^\circ$.

9. The sides of a triangle are a feet, b feet, and $\sqrt{a^2 + ab + b^2}$ feet in length; find its greatest angle.

10. In any triangle, if $\tan \frac{A}{2} = \frac{5}{6}$, and $\tan \frac{B}{2} = \frac{20}{37}$; find $\tan C$.

Show also that, in such a triangle, $a + c = 2b$.

11. DE is a tower on a horizontal plane. $ABCD$ is a straight line in the plane. The height of the tower subtends an angle θ at A , 2θ at B , and 3θ at C . If $AB = 50$ feet, and $BC = 20$ feet, find the height of the tower and the distance CD .

12. Prove the formula $r = \frac{a \sin \frac{B}{2} \sin \frac{C}{2}}{\cos \frac{A}{2}}$; where r is the radius of a

circle inscribed in the triangle ABC .

FURTHER EXAMINATION.

V. PURE MATHEMATICS (I).

(Analytical Geometry, Conic Sections, and Solid Geometry.)

[Full marks may be obtained for about three-fourths of this paper. Great importance is attached to accuracy.]

1. Points A, B, C are taken on three conterminous edges of a cube : prove that the angles of the triangle ABC are all acute.

2. A number of planes pass through a given line : find the locus of the feet of the perpendiculars on them drawn from a given point.

3. Prove that the volume of any pyramid is one-third of the volume of a prism having the same base and altitude.

4. Find the value of c in order that the equation

$$12x^2 + xy - 6y^2 - 29x + 8y = c$$

should represent a pair of right lines.

5. If ω be the angle which a focal chord of a parabola makes with its axis, prove that the length of the chord is $p \operatorname{cosec}^2 \omega$, where p is the parameter of the curve.

6. Being given two tangents to an ellipse, and one of its foci, find the locus of its centre.

7. From any point P on an ellipse PV is drawn perpendicular to its axis major, and produced until NQ is equal to PF , where F is a focus of the ellipse : find the locus of Q .

8. Show that the equation

$$(\alpha^2 + \beta^2 - r^2)(x^2 + y^2 - r^2) = (\alpha x + \beta y - r^2)^2$$

represents a pair of right lines which pass through the point α, β , and touch the circle $x^2 + y^2 = r^2$.

9. In an ellipse prove that the parallelogram formed by joining the extremities of a pair of conjugate diameters has a constant area.

10. Find the equation to the normal at any point on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

If X , Y be the coordinates of the middle point of the portion of the normal which is intercepted between the axes of the ellipse, prove that

$$a^2 X^2 + b^2 Y^2 = \frac{1}{4}(a^2 - b^2)^2.$$

11. Define the polar of a point with respect to a conic, and prove that any right line drawn through a point is divided harmonically by the point, the curve, and the polar of the point.

12. Being given three points on a conic, and one of its foci, find the position of its centre and of its vertices by geometrical construction.

13. Prove that any ellipse can be projected orthogonally into a circle.

Hence show that any triangle can be orthogonally projected into an equilateral triangle.

VI. PURE MATHEMATICS (2).

(Further Questions and Problems on the Subjects of the Qualifying Examination and Theory of Equations.)

[Full marks may be obtained for about four-fifths of this paper. Great importance is attached to accuracy.]

1. If $x + \frac{1}{x} = y$, express $x^6 + \frac{1}{x^6}$ in terms of y .

2. If $\frac{x^2+2}{x^4+1} \equiv \frac{A+Bx}{x^2+x\sqrt{2}+1} + \frac{C+Dx}{x^2-x\sqrt{2}+1}$ for all values of x , find the values of A , B , C , D .

3. Find the sum of the series

$$(1) \quad 1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \dots,$$

$$(2) \quad 1 + 3x + 5x^2 + 7x^3 + \dots,$$

each to n terms.

4. Find, by aid of the Binomial theorem, or otherwise, the limiting value of $\sqrt{x^4+ax^3+bx+c}-\sqrt{x^4+a'x^3+b'x+c'}$ when $x=\infty$.

5. In what time will a given sum of money quadruple itself at 4 per cent. compound interest? (Given $\log 2 = \cdot 30103$, $\log 1\cdot3 = \cdot 11394$.)

6. If $a+b+c=0$, and $x+y+z=0$, prove that

$$(a^2+x^2)(bc-yz)+(b^2+y^2)(ca-zx)+(c^2+z^2)(ab-xy)=0.$$

7. There are 10 tickets, of which 5 are blanks, and the others are marked 1, 2, 3, 4, 5; find the probability of drawing 9 in three trials, the tickets not being replaced.

8. Find the sum of the series

$$\cos a + \cos 3a + \cos 5a + \dots + \cos (2n-1)a.$$

9. In a plane triangle prove that

$$a^3\sin(B-C)+b^3\sin(C-A)+c^3\sin(A-B)=0,$$

10. Find the real and imaginary parts of the expressions

$$\sin(\alpha+\beta\sqrt{-1}) \text{ and } \sec(\alpha+\beta\sqrt{-1}), \text{ where } \alpha \text{ and } \beta \text{ are real.}$$

11. Show that the equation $x^3-4x^2-3x+12=0$ has two roots which are equal, with opposite signs: and solve the equation.

12. Prove that a root of the equation $f'(x)=0$ lies between each adjacent pair of real roots of $f(x)=0$.

Solve the equation $x^4-3x^3+x^2+4=0$, which has a pair of equal roots.

13. If α, β, γ be the roots of the cubic

$$x^3+px^2+qx+r=0,$$

form the equation whose roots are

$$\alpha\beta+\frac{2}{\gamma}, \quad \gamma\alpha+\frac{2}{\beta}, \quad \beta\gamma+\frac{2}{\alpha}.$$

VII. PURE MATHEMATICS (3).

(Differential and Integral Calculus.)

[Full marks may be obtained for about three-quarters of this paper. Great importance is attached to accuracy.]

1. Differentiate the functions

$$\log \left(\frac{x^3 - 1}{x^2 + 1} \right)^{\frac{1}{4}}, \quad \frac{x}{\sqrt{a^2 - x^2}}, \quad (\sin x)^x.$$

2. Find the third differential coefficient of y^2 ; y being a given function of x .

3. The base angles A, B , of an isosceles triangle ACB are infinitely small. Show that in the limit the triangle is equal to $\frac{3}{4}$ of the circular segment ACB .

If α is infinitesimal, show that $2 \sin \alpha + \tan \alpha - 3\alpha$ is an infinitesimal of the fifth order.

4. State Maclaurin's theorem for the expansion of a function.

Expand $\int \left(\frac{x}{1-x} \right) dx$ by powers of x as far as x^3 .

5. Find the value of the vanishing fraction $\frac{x^a - a^x}{x - a}$ when $x = a$.

6. If m, n are positive, and if

$$mx^2 + ny^2 = a^2,$$

find by any method the greatest possible value of $x + y$.

7. Find the equation of the tangent to the curve

$$x^m y^n = a^{m+n}$$

at any point; and show that the portion of the tangent intercepted by the axes is divided in a constant ratio at the point of contact.

8. Find the integrals

$$\int \left(\frac{1-x}{1+x} \right)^2 dx; \quad \int \frac{dx}{\sqrt{1+x \pm x^2}}; \quad \int x^{r-1} \log x \, dx.$$

In the second, either of the signs $\pm$ may be chosen.

9. Find a formula for reducing the integral $\int \cos^n x \, dx$, where n is an integer. When may the integral be easily found without using this formula? Modify the formula so as to give one for the reduction of

$$\int \frac{dx}{\cos^n x}.$$

10. If $z = x^{-r} \int x^{r-1} u \, dx$, prove that

$$x \frac{dz}{dx} + rz = u.$$

11. In the curve $x^m y^n = 1$ the whole area included between the curve and one of its asymptotes [beginning from the point (1, 1)] is infinite, and that between it and the other is finite. Find the value of the latter area.

12. Find the radius of curvature of an ellipse, and show that it is equal to $\frac{b'^3}{ab}$, where b' is the semi-diameter conjugate to that through the point.

VIII. STATICS.

1. Explain what is meant by Graphical Statics. If forces of 5 lbs. and 6 lbs. be represented by the base and altitude of an isosceles triangle, what forces will be represented by the sides?

2. Two weights W_1, W_2 , are connected by a string which hangs over a smooth peg, and motion is prevented by means of a second string, shorter than the first, attached to each weight. Find the tensions of the strings.

3. Enunciate the proposition known as the Parallelogram of Forces, and, assuming its truth for the direction, prove it for the magnitude of the resultant.

A weight of 10 lbs. is suspended by two strings, 7 and 24 inches long, their other ends being fastened to the extremities of a rod whose length is 25 inches. If the rod be held so that the weight hangs immediately below its middle point, find the tensions of the strings.

4. Three forces $5P, 10P$, and $13P$, act in one plane on a particle, the angles between any two of their directions being 120° . Find the magnitude of their resultant.

WOOLWICH ENTRANCE EXAMINATION.

5. Show how to find the resultant of two like parallel forces. A rod 16 inches long rests on two pegs, 9 inches apart, with its centre midway between them. The greatest weights which can be suspended from the two ends of the rod without disturbing the equilibrium are 4 lbs. and 5 lbs. respectively. Find the weight of the rod, and the position of its centre of gravity.

6. Prove that any system of co-planar forces can be reduced to a single force or a single couple.

7. Find the centre of gravity of a triangular lamina.

The base of an isosceles triangle is 4 inches, and the equal sides are each 7 inches: find the distances of its centre of gravity from the angular points of the triangle.

8. Find the magnitude of the least force which will draw a weight W up a rough inclined plane, the coefficient of friction being μ , and the force being supposed to act parallel to the plane.

How much work is done in drawing a load of 6 cwt. up a rough inclined plane whose height is 3 feet and base 20 feet, the coefficient of friction being $\frac{2}{15}$?

9. Assuming the principle of Virtual Velocities, deduce the relation between the power and the weight in the case of the wheel and axle.

10. Find the force required to raise a bar weighing 20 lbs. by means of a system of one fixed and three movable pulleys; each string being attached to the bar, and the weight of the pulleys being neglected. Draw carefully a figure showing this system of pulleys.

11. Prove that if a body be placed on a horizontal plane, it will stand or fall according as the vertical line drawn through its centre of gravity passes within or without the base.

IX. DYNAMICS.

[The measure of the acceleration of gravity may be taken to be 32 when a foot and a second are units of length and time.]

[Great importance will be attached to accuracy.]

1. What is meant by *uniform velocity*, and how is it measured? What by *uniform acceleration*, and how measured?

A body starting with a velocity of 10 feet per second describes 11 feet in one second. If the acceleration on it be uniform, find its value.

2. A steamer is going due N. with velocity v ; the smoke from the chimney points θ degrees South of E. If the wind is due W., find its velocity.

3. If the measure of an acceleration is f , what change in this measure is produced by taking double the unit of time as the new unit? Prove the result.

If the measure of gravity-acceleration be 1, what is necessary as to the units of space and time?

4. Two bodies start together from rest, from one point, in two diverging lines, each moving with a given acceleration. Show that their velocity of separation is uniformly accelerated during the motion.

5. How is *angular velocity* measured when uniform? A body revolves round an axis with a uniform angular velocity ω ; how many complete revolutions does it make per minute.

A point moves so that its angular velocities round two fixed points are always equal and in the same sense; find the path it describes.

6. A perfectly elastic ball strikes another equal ball moving in the same direction. Show from the principles of impact, and without assuming any formulae, that the balls exchange velocities.

7. Given the initial velocity and elevation of a projectile, construct geometrically its position, and the direction in which it is moving, at any given moment.

From your construction (or otherwise) show that if any number of projectiles are thrown together from one point, whatever be their initial motions, their directions of motion at any instant all pass through one point.

8. A vertical hollow cylinder, 3 feet in circumference, is revolving uniformly round its axis, making one turn per second. A heavy particle is let drop from a point at the mouth of the cylinder, so as just to touch the inside surface during its fall and leave a trace upon it. If the cylinder be afterwards unrolled, what form will the trace present?

9. A body moves uniformly with given velocity in a circle. Is there necessarily any force acting on the body? If so, state its magnitude and direction at any moment.

A smooth, hollow sphere is revolving with uniform angular velocity ω round a vertical diameter. Show that a heavy particle placed inside will only remain, resting against the side of the sphere, at one particular level.

If the angular velocity ω be less than a certain limit, show that the particle will remain at the lowest point of the sphere.

10. State the principle of work with regard to the motion (1) of a particle acted on by force, (2) of a rigid body. If a body is at rest, and any forces are applied to it, and it moves under the action of those forces, until it arrives at another position where it is also at rest, what can be inferred as to the action of the forces?

A uniform rectangular block $ABCD$ stands on its base AD on a rough floor. It is pulled at C by a horizontal force just great enough to begin to turn it round the corner D . If this same force continues to pull it horizontally at C till the block has turned through an angle θ , and then ceases, prove that the block will have acquired sufficient momentum to cause it just to overturn round D , provided $\sin \theta = \tan \frac{1}{2}\alpha$, where $\alpha < BDC$.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1887.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. The angles which one straight line makes with another straight line on one side of it, either are two right angles, or are together equal to two right angles.

The straight lines bisecting the angles at the base BC of an isosceles triangle ABC intersect in D . Show that AD produced bisects the angle BDC .

2. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are equal to two right angles.

Describe an isosceles triangle in which each of the angles at the base is one-fourth of the vertical angle.

3. Distinguish between a *Problem* and a *Theorem*, and explain what is meant by the *converse* of a Theorem.

Enunciate carefully, and prove, the converse of the following theorem :
“Triangles on equal bases and between the same parallels are equal.”

4. If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the two parts.

Prove that, of all right-angled parallelograms having the same perimeter, the square has the shortest diagonals.

5. In every triangle the square on the side subtending an acute angle is less than the squares on the sides containing that angle by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle and the acute angle.

6. The diameter is the greatest straight line in a circle; and, of all others, that which is nearer to the centre is always greater than one more remote; and the greater is nearer to the centre than the less.

7. In a circle the angle in a semicircle is a right angle, but the angle in a segment greater than a semicircle is less than a right angle, and the angle in a segment less than a semicircle is greater than a right angle.

Two circles, whose centres are O, O' , touch each other in the point A , and are met by a common tangent in the points B, C , respectively. $BOD, CO'E$, are diameters of the two circles. Show that BE, CD , intersect in A .

8. Inscribe a circle in a given square.

Two squares are described, one circumscribing and the other inscribed in a given circle. Prove that the diameter of the circle inscribed in the smaller square is equal to the radius of the circle described about the larger square.

9. How does Euclid test whether four magnitudes are proportionals? Find a fourth proportional to three given straight lines.

10. If four straight lines be proportionals, the rectangle contained by the extremes is equal to the rectangle contained by the means; and, if the rectangle contained by the means be equal to the rectangle contained by the extremes, the four straight lines are proportionals.

Describe a rectangle so that its sides are in a given ratio, and its area is equal to a given square.

11. Parallelograms about the diameter of any parallelogram are similar to the whole parallelogram and to one another.

Show that either of the complements is a mean proportional between the two parallelograms about the diameter.

II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—Great importance is attached to accuracy. Candidates are expected to use Arithmetical methods of solution.]

1. Multiply 909 by 990, and explain your method of work.
2. What is the value of 6 oz. 12 dwts. of gold when 1 oz. 13 dwts. cost £4. 18s. 10½d.?
3. State and explain the rule for division of fractions. Divide $33\frac{3}{8}$ by $7\frac{3}{8}$, and bring the result to its lowest terms.
4. If the sum of the digits of a number be divisible by 9, prove that the number itself is also divisible by 9.
5. Divide '0053 by 2'5, and 53 by '0025. Explain your rule for pointing in the quotient in each case.
6. Reduce £2. 15s. 6¾d. to the decimal of a pound; and find the difference between '625 and '0625 of a cwt.
7. Reduce '007648 to a vulgar fraction; and add together '125, 4'163, 1'7143, and 2'54 without reducing them to fractions.
8. If the carriage of 150 feet of wood, that weighs 3 stone per foot, costs £3 for 40 miles, how much will the carriage of 54 feet of marble, weighing 8 stone per foot, cost for 25 miles?
9. Find, by Practice, the cost of 92 tons 7 cwts. 3 qrs. 12 lbs. at £1. 12s. 7d. per ton.
10. State and explain the rule of pointing in the extraction of a square root. Extract the square root of 197'96492 to 7 places of decimals.
11. Find the solid content of a block of marble 3 ft. 8 in. 4' long, 1 ft. 7 in. 6' broad, and 1 ft. 4 in. 9' thick (by duodecimals). Express your result in cubic feet and inches.
12. In how many years will £768. 17s. 6d. amount to £1230. 4s. at 5 per cent. simple interest?

WOOLWICH ENTRANCE EXAMINATION.

13. A person invests £1842. 15s. in 4 per cents. at $102\frac{3}{8}$; he afterwards sells out at 105, and re-invests at 126 in 5 per cents.; find his change of income.

14. Two watches are together at 12 o'clock. One gains $75''$ per hour, and the other loses $45''$. When will they be together again at 12?

15. If A , B , and C could reap a field in 18 days; B , C , and D in 20 days; C , D , and A in 24 days; and D , A , and B in 27 days; in what time would it be reaped by them all together?

16. If a grocer gains 10 per cent. by selling tea at 2s. 3d. per lb., what will he gain per cent. by selling it at 2s. 9d.?

17. Define a Logarithm. Find the log of 81 to base 3, and of 3 to base 81. Given $\log 234 = 2.3692159$, write down $\log 234000$, $\log 23.4$, $\log .0234$.

18. Find, with the aid of the logarithm tables supplied, a 4th proportional to 1.3046, .01042, and 2.375.

19. Given $\log 367200 = 5.5649027$, and $\log 367100 = 5.5647844$: find the number whose log is 5.5648815.

20. Find, with the aid of the tables, the amount of £1,000 in 100 years at 5 per cent.—(1) Simple interest. (2) Compound interest.

21. A hare starts to run, when a dog is 44 yards off, at 12 miles per hour. After half a minute the dog sees her, and pursues at 16 miles per hour. How soon will he catch her?

22. Find the present value of an annuity of £300 payable every year for 3 years at 5 per cent. per annum simple interest.

[Tables of logarithms to seven figures are supplied.]

III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—Great importance is attached to accuracy.]

1. Divide $x^3(a+1) - xy(x-y)(a+b) - y^3(b-1)$ by $x^2 - xy + y^2$.

2. Prove that, if

$$y - z = ax,$$

$$z - x = by,$$

$$x - y = cz,$$

then

$$abc + a + b + c = 0.$$

3. Prove the rule for finding the Least Common Multiple of two algebraic expressions.

Find the Greatest Common Measure of

$$x^4 + 4x^3y - 21x^2y^2 + 10xy^3 - y^4 \quad \text{and} \quad x^4 + 12x^3y + 33x^2y^2 - 12xy^3 + y^4.$$

4. Simplify the following expressions :

$$(1) \left(\frac{x^2}{y^2} - 1 \right) \left(\frac{x}{x-y} - 1 \right) + \left(\frac{x^3}{y^3} - 1 \right) \left(\frac{x^2 + xy}{x^2 + xy + y^2} - 1 \right);$$

$$(2) \frac{x^2 - 7xy + 12y^2}{4x^2 - 11xy - 3y^2} - \frac{2x^2 + 7xy - 4y^2}{8x^2 - 6xy + y^2}.$$

5. Prove that the greater of the two fractions

$$\frac{\sqrt{3} + \sqrt{2}}{\sqrt{2} + \sqrt{2} + \sqrt{3}}, \quad \frac{\sqrt{3} - \sqrt{2}}{\sqrt{2} - \sqrt{2} - \sqrt{3}},$$

exceeds the less by $2 - \sqrt{2}$.

6. Extract the square root of

$$4 \left(\sqrt[3]{\frac{a^2}{c^2}} + \sqrt[3]{\frac{c^2}{a^2}} - \sqrt[3]{\frac{a}{c}} - \sqrt[3]{\frac{c}{a}} \right) + 9.$$

7. If α, β , be the roots of the quadratic $x^2 + px + q = 0$, prove that the equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$ is

$$qx^2 + p(1+q)x + (1+q)^2 = 0.$$

8. Solve the equations :

$$(1) \quad \frac{2x-y}{10a+3b} = \frac{x-3y}{4b} = \frac{y+b}{2a};$$

$$(2) \quad \frac{1}{2x-5a} + \frac{5}{2x-a} = \frac{2}{a};$$

$$(3) \quad x - \sqrt{x^2 - 1} = \frac{1}{2(x+1)}.$$

9. A and B run a race; B has 50 yards start, but A runs 20 yards while B runs 19: what must be the length of the course, that A may come in a yard ahead of B ?

10. What is meant by the statement that one quantity varies directly as another?

If the number of passengers by an excursion train vary directly as the square of the number of degrees of temperature above 50° Fahrenheit, and if there be 405 passengers when the temperature is 59° , how many will there be when the temperature is 56° ?

11. Assuming the formula for the number of arrangements of n different things taken r together, prove the formula for the number of combinations of n things taken r together.

A man has 10 friends: how many different parties could he make by inviting them six together?

12. Show that in the expansion of $(1+x)^n$, where n is a positive integer, the coefficients of terms equidistant from the beginning and end are equal.

Write down the middle term in the expansion of $\left(x - \frac{1}{x}\right)^{2n}$ and the coefficient of x^r in the expansion of

$$(1+2x+3x^2+\dots)^2.$$

IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy. Logarithm tables are supplied.]

1. Define the sine, cosine, and tangent of an angle, and express each of the three trigonometrical ratios in terms of the other two.

Find the sines and cosines of all the angles in the first four quadrants whose cosine is equal to $\cos 135^\circ$.

2. Prove that $\sin(A+B) = \sin A \cos B + \cos A \sin B$, and deduce from it corresponding formulæ for $\cos(A-B)$ and $\cos 2A$.

Prove that $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0$.

3. Prove that

$$\tan\left(45^\circ + \frac{A}{2}\right) = \left(\frac{1 + \sin A}{1 - \sin A}\right)^{\frac{1}{2}}$$

and

$$\cos^3 2\theta + 3 \cos 2\theta = 4 (\cos^6 \theta - \sin^6 \theta).$$

4. Find the values of $\tan^{-1}\sqrt{3}$ and $\cos 105^\circ$.

Construct the acute angle whose cosine is equal to its tangent.

5. Prove that

$$\cos^{-1}x = \sin^{-1}\sqrt{\frac{1-x}{2}} + \cos^{-1}\sqrt{\frac{1+x}{2}}.$$

6. Express $\sin 4A$ in terms of $\tan A$; and solve the equation

$$\cos 3\theta + \sin \theta = 0.$$

7. Prove that in any triangle $c = a \cos B + b \cos A$.

Hence show that

$$s = a \cos^2 \frac{B}{2} + b \cos^2 \frac{A}{2} = b \cos^2 \frac{C}{2} + c \cos^2 \frac{B}{2} = c \cos^2 \frac{A}{2} + a \cos^2 \frac{C}{2};$$

and deduce the equation

$$\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}.$$

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8. The base of a triangle being seven feet, and the base angles $129^{\circ} 23'$ and $38^{\circ} 36'$, find the length of the shorter side.

9. Prove that in any triangle

$$\frac{a+c}{b} = \frac{\cos \frac{A-C}{2}}{\sin \frac{B}{2}}.$$

Hence show that, if C is a right angle

$$\tan \frac{A}{2} = \frac{a-b+c}{a+b+c}.$$

10. A flagstaff a feet high is on a tower $3a$ feet high: prove that, if the observer's eye is on a level with the top of the staff, and the staff and tower subtend equal angles, the observer is at a distance $a\sqrt{2}$ from the top of the staff.

11. If S and S' are the areas of a given triangle and the triangle formed by joining the points of contact of its inscribed circle, prove that

$$\frac{S'}{S} = \frac{2(s-a)(s-b)(s-c)}{abc}.$$

[Logarithm tables to seven figures are supplied.]

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

(Analytical Geometry, Conic Sections, and Solid Geometry.)

*[Full marks may be obtained for about three-fourths of this paper.
Great importance is attached to accuracy.]*

1. From a point in space perpendiculars are let fall, one on a plane, another on a straight line lying in that plane. Prove that the line joining the feet of the perpendiculars is perpendicular to the straight line.

2. Prove that in a trihedral angle any two of the dihedral angles containing it are together greater than the third.

If A, B, C, D are four points not in one plane, prove that the four angles of the quadrilateral $ABCD$ are together less than four right angles.

3. A circle in space is projected orthogonally on any plane. Show that the projection is an ellipse, and find its axes.

4. Three lines in space, OA, OB, OC , are mutually at right angles. Their lengths being a, b, c , express the area of the triangle ABC in its simplest form.

5. From a fixed point A on a given line any number of lengths, AP , are measured along the line; and at P a perpendicular PQ is erected, whose length in each case represents the square root of the number represented by AP . Trace the locus of Q , and give the position of its focus.

6. Show that $x + y = \sqrt{a^2 + b^2}$ represents a tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

and find the point of contact.

7. A variable circle always passes through two fixed points A, B . Find the locus of a point on it where the tangent is perpendicular to AB .

8. The vertex O of a parabola is joined with any point P on the curve; and PQ is drawn at right angles to OP , meeting the axis in Q . Show that the projection of PQ on the axis is always equal to the latus rectum.

9. If, in the equation $y = mx + c$, c is given, but not m , how far is the position of the straight line determined?

Give an equation which will include all straight lines passing through the intersection of two given straight lines.

10. Two points (xy) , $(x'y')$, subtend a right angle at the origin. What relation exists among their coordinates?

What loci are represented by the equations

$$(1) \ x^2 + xy + y^2 = 0, \quad (2) \ x^3 - xy^2 = 0, \quad (3) \ (x^2 - a^2)^2 + (y^2 - b^2)^2 = 0.$$

11. Prove geometrically that the tangent to a parabola is equally inclined to the focal radius vector and to the diameter.

A parabola touches two given straight lines at two given points. Find its focus by geometrical construction.

12. Assuming that the sum of the focal distances of any point on an ellipse is constant, and that the tangent is equally inclined to them; prove *geometrically* that:—

(1) The locus of the foot of the perpendicular from a focus on any tangent is a circle.

(2) The rectangle of the perpendiculars from the two foci on any tangent is constant.

VI. PURE MATHEMATICS (2).

(Further Questions and Problems on the Subjects of the Qualifying Examination, and Theory of Equations.)

[Full marks may be obtained for about four-fifths of this paper. Great importance is attached to accuracy.]

1. If it is possible to find values for x , y , z , to satisfy the equations

$$x^2 + y^2 = 2axy,$$

$$y^2 + z^2 = 2byz,$$

$$z^2 + x^2 = 2czx,$$

show that

$$a^2 + b^2 + c^2 = 2abc + 1.$$

2. If x be any positive quantity, and n any integer, prove that

$$\left(\frac{x+1}{n+1}\right)^{n+1} > \left(\frac{x}{n}\right)^n,$$

being given the theorem that, if a positive number be divided into any given number of parts, the product of the parts is greatest when they are equal.

3. Express $y = \sqrt{1+x^2}$, (1) in a series of ascending powers of x , (2) of descending powers.

Give an approximate value for y when x is a large number.

4. Prove in any manner that $n^4 - 1$ is divisible by 5 if n be any integer not divisible by 5.

Show that, n being any integer, $n^5 - n$ is divisible by 30.

5. If n straight lines are drawn at random on a plane, how many points of intersection will there be?

If n pairs of lines are drawn, each pair being parallel to one another, how many intersections?

6. Three cards are drawn at random from a pack of 52. Find the chance that all three are from different suits.

7. State the series for $\log(1+x)$ by ascending powers of x .

Prove also that

$$\log(1+x) = \frac{x}{1+x} + \frac{1}{2} \left(\frac{x}{1+x} \right)^2 + \frac{1}{3} \left(\frac{x}{1+x} \right)^3 + \text{etc.}$$

8. Find the three roots of the cubic $4x^3 - 15x^2 + 12x + 4 = 0$.

Prove that, for all positive values of x except 2,

$$x^3 + 3x + 1 > \frac{15}{4}x^2.$$

9. If $\tan^2 \theta = \frac{b}{a}$, find, in terms of a and b , the simplest form for the value of $\frac{a}{\cos \theta} + \frac{b}{\sin \theta}$.

10. What are the factors of $1 - 2x \cos \theta + x^2$?

Find the sum of the series

$$\sin \theta + x \sin 2\theta + x^2 \sin 3\theta + \text{etc.}$$

WOOLWICH ENTRANCE EXAMINATION.

11. State the formula for the tangent of the sum of any number of angles.

If n be odd, express by Trigonometry the n roots of the equation

$$nx - \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} x^3 + \frac{n(n-1)(n-2)(n-3)(n-4)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} x^5 - \text{etc.} = 0.$$

12. A long building in front of us is found to subtend an angle θ ; on walking a distance d perpendicularly towards it, the length of building subtends an angle ϕ ; on continuing a distance e in the same direction, the building is reached. Find its length.

VII. PURE MATHEMATICS (3).

(Differential and Integral Calculus.)

*[Full marks may be obtained for about three-fourths of this paper.
Great importance is attached to accuracy.]*

1. Starting from first principles, find the values of $\frac{du}{dx}$, (1) when $u = x^m$, for all real values of m ; (2) when $u = \log x$.

2. Explain what is meant by different orders of infinitesimals. If a side of a regular polygon inscribed in a circle be a very small magnitude of the first order in comparison with the radius, show that the difference between the circumference of the circle and the perimeter of the polygon is a small magnitude of the second order.

3. If $a \tan x = b \tan y$, express $\frac{dx}{dy}$ in terms of y and of the constants a, b .

4. If $p^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$, prove that $p + \frac{d^2 p}{d\theta^2} = \frac{a^2 b^2}{p^3}$.

5. Explain Lagrange's theorem on the limits of Taylor's theorem. Give an instance in which the expansion according to Taylor's theorem is shown by Lagrange's theorem to be inapplicable.

6. Find an expression for the length of the subtangent at any point on the curve

$$xy^2 = a^2(a - x),$$

and find the point at which the subtangent has a maximum length.

7. Define the pedal of a curve with respect to any point.

Find the equation to the pedal of the curve

$$r^m = a^m \sin m\theta.$$

8. In the curve

$$(a^2 - 2ay)(3y^2 - x^2) = 2(x^4 - y^4)$$

find the position of the asymptotes, and also the direction of the tangents at the origin.

9. Find the values of the following integrals :

$$\int \sin^2 x \, dx, \quad \int \log x \, dx, \quad \int \frac{dx}{x^2 + x - 6}.$$

10. Find the values of the definite integrals :

$$\int_0^{\frac{\pi}{2}} \cos x \, dx, \quad \int_0^{\frac{\pi}{2}} x \sin x \, dx, \quad \int_0^1 \frac{dx}{1 + x + x^2}.$$

11. Show how to express the area of a curve whose equation is given in polar coordinates.

Find the area of a loop of the curve $r = a \cos 3\theta$.

12. Find the length of an arc, measured from the vertex, of the semi-cubical parabola $ay^2 = x^3$.

VIII. STATICS.

1. Enunciate and prove the proposition known as the Triangle of Forces. Construct geometrically the directions of two forces $2P$ and $3P$ which make equilibrium with a force $4P$ whose direction and point of application are given.

2. Forces act in one plane on a particle; show how to find the magnitude and direction of their resultant.

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$ABCD$ is a square; forces of 1 lb., 6 lbs., and 9 lbs. act in the directions AB , AC , and AD respectively: find the magnitude of their resultant correct to two places of decimals.

3. A light triangular frame ABC stands in a vertical plane, C being uppermost, on two supports A , B , in the same horizontal line, and a weight of 18 lbs. is suspended from C . If $AB = AC = 6$ feet, and $BC = 5$ feet, find the pressures on the supports.

4. If four forces in equilibrium be parallel, two and two, prove that they form two unlike couples of equal moment.

A uniform ladder 13 feet long rests with one end against a smooth vertical wall, and the other on a rough horizontal plane at a point 6 feet from the wall. Find the friction between the ladder and the ground, the weight of the ladder being 56 lbs.

5. Find the centre of gravity of a pyramid, and show that it coincides with that of four equal particles placed at the angular points.

6. A triangular lamina ABC , obtuse-angled at C , stands with its side AC in contact with a table: show that the least weight which, suspended from B , will overturn it is $\frac{1}{3}W \frac{a^2 + 3b^2 - c^2}{c^2 - a^2 - b^2}$, where W = the weight of the triangle.

Interpret the above if c^2 be $> a^2 + 3b^2$.

7. A light rod rests wholly within a smooth hemispherical bowl of radius r , and a weight W is clamped on to the rod at a point whose distances from the ends are a , b . Show that, if θ be the inclination of the rod to the horizon in the position of equilibrium, then $\sin \theta = \frac{a-b}{2\sqrt{r^2-ab}}$; and find the pressures between the rod and the bowl.

8. Investigate the relation between the power and the weight in the case of a smooth screw.

What must be the length of the power arm of a screw having six threads to the inch in order that the mechanical efficiency may be 216?

9. State the laws of limiting friction.

The poles supporting a lawn-tennis net are kept in a vertical position by guy ropes, one to each pole, which pass round pegs 2 feet distant from the poles. If the coefficient of limiting friction between the ropes and pegs be $\frac{4}{3}$, show that the inclination of the latter to the vertical must not be less than $\tan^{-1} \frac{2}{11}$, the height of the poles being 4 feet.

10. Explain the terms *virtual velocity* and *virtual moment* of a force.

A uniform beam AB of weight W , movable in a vertical plane about a hinge at A , is kept in a given position by a force P applied at the end of a string PCB passing over C , a pulley vertically above A , and at a distance $AC = AB$. Show that, if the system be slightly displaced owing to a small change in P —

$$P \times P\text{'s virtual velocity} + W \times W\text{'s virtual velocity} = 0.$$

IX. DYNAMICS.

[The measure of the acceleration of gravity may be taken to be 32 when a foot and a second are the units of length and time.]

[Great importance will be attached to accuracy.]

1. Compare the velocities of the extremities of the hour, minute, and second-hands of a watch, their lengths being .48, .8, and .24 inches respectively.

2. Enunciate and prove the proposition known as the Parallelogram of Velocities.

Two points describe the same circle in such a manner that the line joining them always passes through a fixed point: show that at any moment their velocities are proportional to their distances from the fixed point.

3. Find the space described in time t by a particle projected with velocity u , subject to a constant acceleration f in the direction of its motion.

If a , b , c , be the spaces described in the p th, q th, and r th seconds, prove that $a(q-r) + b(r-p) + c(p-q) = 0$.

4. If the acceleration due to gravity be taken as the unit acceleration, and the velocity generated in one minute as the unit of velocity, find the unit of length.

5. Two weights, P , Q , are connected by a weightless string which passes over a smooth pulley: find the acceleration and the tension of the string.

Two strings pass over a smooth pulley; on one side they are each attached to a weight of 5 lbs., and on the other to weights of 3 lbs. and 4 lbs. respectively; find their tensions.

6. A body is projected in a given direction with a given velocity; find the range on a given inclined plane passing through the point of projection.

The angular elevation of an enemy's position on a hill h feet high is β : show that, in order to shell it, the initial velocity of the projectile must not be less than $\sqrt{gh(1 + \operatorname{cosec} \beta)}$.

7. A ball of mass m moving with velocity u impinges directly upon a ball of mass m' moving with velocity u' : find the velocities of the balls after impact, e being the coefficient of elasticity.

Two equal marbles A, B , lie in a horizontal circular groove at opposite ends of a diameter. A is projected along the groove, and after a time t impinges on B : show that a second impact will take place after a further interval $\frac{2t}{e}$.

8. A particle slides down a smooth curve in a vertical plane under the action of gravity: find the velocity of the particle in any position.

A bead slides on a wire bent into the form of a parabola whose axis is vertical and vertex upwards; if the bead be just displaced from its position of equilibrium, then at any subsequent time its velocity will vary as its distance from the axis.

9. A particle weighing $\frac{1}{2}$ oz. rests on a horizontal disc and is attached by two strings 4 feet long to the extremities of a diameter. If the disc be made to revolve 100 times a minute about a vertical axis through its centre, find the tension of each string.

10. A train weighing 200 tons is running at 40 miles an hour down an incline of 1 in 120: find the resistance necessary to stop the train in half a mile.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1888.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed; but the method of proof must be geometrical. Great importance will be attached to accuracy.]

1. If two triangles have two sides of the one equal to two sides of the other, each to each, and have likewise their bases, or third sides, equal; the angle which is contained by the two sides of the one is equal to the angle contained by the two sides, equal to them, of the other.

ACB , ADB are two triangles, on the same base AB and on the same side of it; AC is equal to BD , and AD to BC ; and AD , BC intersect in O : prove that the triangles OAB , OCB are each isosceles.

2. If a straight line falls upon two parallel straight lines, it makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle on the same side of the line, and the two interior angles on the same side of it together equal to two right angles.

AB and CD are any two diameters of a circle; prove that, if C and D be joined to B , the straight lines CB , DB will bisect the angles made with AB by a straight line through B parallel to CD .

3. If a parallelogram and a triangle are on the same base and between the same parallels, the parallelogram is double of the triangle.

Construct the greatest triangle which has two of its sides equal to two given straight lines each to each.

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4. If a straight line is bisected and produced to any point, the rectangle contained by the whole line thus produced and the part produced, together with the square on half the line bisected, is equal to the square on the line which is made up of the half and the part produced.

Find a point D in AB produced so that the rectangle AD, DB shall be equal to the square on a given straight line which is greater than half AB .

5. If a straight line is divided into two equal parts, and also into two unequal parts, the squares on the unequal parts are together double of the square on half the line, and of the square on the part between the points of section.

6. The angles in the same segment of a circle are equal to one another.

If two segments of circles have a common chord AB , and any points P and Q are taken on their arcs, the locus of the point of intersection of the bisectors of the angles PAQ, PBQ is another segment of a circle on the same chord AB .

7. Upon a given straight line describe a segment of a circle, which shall contain an angle equal to a given rectilineal angle.

8. Describe a circle about a given triangle.

Show that, if the centre of the circle described about a triangle coincides with the centre of the circle inscribed in it, the triangle must be equilateral.

9. The sides about the equal angles of equiangular triangles are proportionals, those sides being homologous which are opposite to the equal angles.

A and B are the points of intersection of two circles; CAD is a chord through A ; and BC, BD are drawn: prove that BC is to BD as the diameter of the circle ABC is to that of the circle ABD .

10. Similar triangles are to one another in the duplicate ratio of their homologous sides.

Of two equilateral and equiangular hexagons, one is inscribed in a circle and the other is described about the same circle: prove that the areas of the hexagons are to one another in the ratio of 3 : 4.

11. In any right-angled triangle, a rectilineal figure described on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle.

Explain the term "similarly described."

II. ARITHMETIC.

(Including the use of Common Logarithms.)

[N.B.—Great importance is attached to accuracy. Candidates are expected to use Arithmetical methods of solution.]

1. Divide 10101255 by 2185, and explain the process.

2. Give a definition of Multiplication which shall include the multiplication of one fraction by another.

Simplify the expression $\frac{3\frac{1}{2}}{7\frac{1}{4}} \times \frac{17\frac{2}{5}}{51\frac{1}{11}}$.

3. Find the Greatest Common Measure, and the Least Common Multiple of the numbers 15496 and 12665, and show, by general reasoning, that these four numbers form a proportion.

4. Multiply 4'327615 by '003248, and divide '292262016 by 32'7648.

5. Find the rent of a piece of ground covering 14 acres 16 poles 22 sq. yards 6 sq. feet at £3. 0s. 6d. per acre.

6. Explain what is meant by a Recurring Decimal, and find the vulgar fractions equivalent to the decimals, '123, 11'1234, and '0034568.

7. Reduce $\frac{3}{8}$ of 4 guineas to the fraction of £7, and £3. 17s. 2½d., to the decimal of £4 to eight places of decimals.

8. Find the Square Root of 199'6569.

When a regiment of 962 men is drawn up in a solid square, one man is left out; find the number of men in the face of the square.

9. A can do a piece of work in 11 days, B in 20 days, and C in 55 days; how soon can the work be done if A is assisted by B and C on alternate days?

10. Find the cost of carpet 2 feet 3 inches wide for a room 20 ft. 3 in. long by 13 ft. 4 in. wide, at 5s. a yard.

11. If the simple interest on £4373. 6s. 8d. for 1½ years be £246, what is the rate per cent.?

WOOLWICH ENTRANCE EXAMINATION.

12. Find the discount on £51. 15s. 10d., due $4\frac{1}{2}$ years hence, at 3 per cent., simple interest.

13. A gravel walk 6 feet wide runs round a grass-plot 60 feet long and 40 feet wide. If gravel is 3s. per cubic yard, find the cost of a coat of gravel on the path 3 inches deep.

14. A person transfers £4865 from the $3\frac{1}{2}$ per cents. at $93\frac{3}{4}$ to the 3 per cents. at $87\frac{1}{2}$. Find the change in his income.

15. Explain what is meant by the characteristic of a logarithm, and state the rule for finding the characteristic of the logarithms to base 10 of the numbers less than unity.

16. Find the logarithms to the base 2 of

$$64, \frac{1}{64}, \text{ and } \sqrt[3]{32}.$$

17. Find from the tables the logarithms of 35726 and 357'26437.

18. Find from the tables the number whose logarithm is 4'9220534.

19. Employ the tables to find the value of the product of 52'4574 by 3'78472.

20. Employ the tables to find the mean proportional between 33'549 and 44'642.

21. Employ the tables to find the value of the expression

$$(5'7432)^{1'246}.$$

22. A tricycle, going at the rate of 5 miles an hour, passes a milestone, and 14 minutes afterwards, a bicycle, going in the same direction at the rate of 12 miles an hour, passes the same milestone; find when and where the bicycle will overtake the tricycle.

III. ALGEBRA.

(Including Equations, Progressions, Permutations and Combinations, and the Binomial Theorem.)

[N.B.—Great importance is attached to accuracy.]

1. Find the value of

$$x^4 - 5x^3 - 12x^2 - 13x - 7 \text{ when } x = -\frac{1 + \sqrt{-3}}{2}.$$

2. Find the factors of

$$(a+b+c)^2 - a^2 + b^2 - c^2 \text{ and of } x^5 - y^5 - (x-y)^5.$$

3. Reduce to its lowest terms

$$\frac{2x^4 + 17x^3 + 30x^2 + 8x - 5}{x^4 + 4x^3 - 18x^2 - 29x - 10}.$$

4. Simplify

$$(i.) \frac{6x^2 - 5xy - 6y^2}{14x^2 - 23xy + 3y^2} - \frac{15x^2 + 8xy - 12y^2}{35x^2 + 47xy + 6y^2}.$$

$$(ii.) \frac{c}{ab(b-c)(c-a)} + \frac{b}{ac(a-b)(b-c)} + \frac{a}{bc(a-b)(c-a)}.$$

5. Apply the process for extracting the square root to find
- m
- and
- n
- when
- $x^4 + ax^3 + mx^2 + cx + n$
- is a complete square.

6. If the roots of the equation

$$\left(1 - q + \frac{p^2}{2}\right)x^2 + p(1+q)x + q(q-1) + \frac{p^2}{2} = 0$$

are equal, show that $p^2 = 4q$.

7. Solve the equations

$$(a) \frac{5}{x+10} + \frac{8}{x+4} = \frac{13}{x+7}.$$

$$(b) \sqrt{2x+6} - \sqrt{x-1} = 2.$$

$$(c) \frac{a^2(x-b)}{a-b} + \frac{b^2(x-a)}{b-a} = x^2.$$

8. Find the value of

$$\frac{(4 + \sqrt{15})^{\frac{3}{2}} + (4 - \sqrt{15})^{\frac{3}{2}}}{(6 + \sqrt{35})^{\frac{3}{2}} - (6 - \sqrt{35})^{\frac{3}{2}}}.$$

9. The metal of a solid sphere, radius
- r
- , is made into a hollow sphere whose internal radius is
- r
- ; required its thickness. [The volume of a sphere, radius
- r
- , is
- $\frac{4}{3}\pi r^3$
- .]

10. Two rectangular lawns have the same area (
- a^2
-), but the perimeter of the one is one-fourth longer than that of the other, which is a square; required its dimensions.

WOOLWICH ENTRANCE EXAMINATION.

11. If the sum of the first p terms in an A.P. = 0 the sum of the next q terms = $\frac{-a(p+q)q}{p-1}$.

12. Is the coefficient of x^r in the expansion of $(1-x)^{-n}$ equal to the number of combinations of $(n+r)$ things taken r together?

If not, amend the proposition, and prove it.

IV. PLANE TRIGONOMETRY.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy.]

1. Define the circular measure of an angle.

Taking $\pi = \frac{2}{7}$, calculate the number of degrees, minutes, and seconds in the unit of circular measure.

2. Prove geometrically that $\cos 2a = \cos^2 a - \sin^2 a$.

Find the value of $\cos 15^\circ$, and prove that

$$(\sec 15^\circ + \operatorname{cosec} 15^\circ)^2 = 24.$$

3. Write down a single expression for all the angles which have the same cotangent as the angle a ; and give, in a single expression for θ , the complete solution of the equation $3 \tan^2 2\theta = 1$.

4. Prove the following formulæ

$$\begin{aligned}\cos^2 \frac{1}{2} a (1 + \tan \frac{1}{2} a)^2 &= 1 + \sin a, \\ 4(\cos^3 10^\circ + \sin^3 20^\circ) &= 3(\cos 10^\circ + \sin 20^\circ),\end{aligned}$$

and express $4 \cos a \cos \frac{a}{2} \cos \frac{5a}{2}$ as the sum of four cosines.

5. Determine x from the equation $\sin \cot^{-1} \frac{1}{2} = \tan \cos^{-1} \sqrt{x}$; and prove that if $\alpha = \tan^{-1} \frac{1}{7}$, $\beta = \tan^{-1} \frac{1}{3}$, then $\cos 2\alpha = \sin 4\beta$.

6. ABC is an isosceles triangle, right angled at C . D is the middle point of AC . Prove that DB divides the angle B into two parts whose cotangents are as 2 : 3.

7. Prove that in any triangle $c = a \cos B + b \cos A$.

From this and the two corresponding formulæ

$$a = b \cos C + c \cos B, \quad b = c \cos A + a \cos C$$

deduce the "Rule of Sines."

8. Prove that in any triangle

$$\frac{b-c}{a} \cos^2 \frac{A}{2} + \frac{c-a}{b} \cos^2 \frac{B}{2} + \frac{a-b}{c} \cos^2 \frac{C}{2} = 0.$$

and if the sines of the angles are as 13 : 14 : 15 find the ratio of the cosines.

9. Obtain the expression for the area of any triangle in terms of the sides.

If the sides of a triangle are in Arithmetical Progression and its area = $\frac{3}{8}$ ths of the area of an equilateral triangle of the same perimeter, prove that its sides are in the ratio 3 : 5 : 7.

10. Given two sides of a triangle and the included angle, obtain a formula, adapted to logarithmic computation, which will enable us to determine the other two angles.

Given $a = 2.7402$, $b = .7401$, $C = 59^\circ 27' 5''$, solve the triangle completely, with the aid of the logarithm tables supplied.

11. The extremity of the shadow of a flagstaff 6 feet high, standing on the top of a regular pyramid on a square base, just reaches a side of the base and is distant 56 feet and 8 feet from the extremities of that side. If the height of the pyramid be 34 feet, find the sun's altitude.

[Logarithm Tables to seven figures were supplied.]

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

(Analytical Geometry, Conic Sections, and Solid Geometry.)

[Full marks may be obtained for about four-fifths of this paper. Great importance is attached to accuracy.]

1. When are two non-intersecting straight lines said to be at right angles?

If the two edges AB , CD of a pyramid be at right angles, show that

$$AC^2 + BD^2 = AD^2 + BC^2.$$

2. Prove that the plane angles which contain any solid angle are together less than four right angles.

3. Define a right circular cone, and show that if a plane be drawn through any two generating lines which intersect at an angle 2β , it will touch a fixed cone whose vertical angle $= 2 \cos^{-1} \{ \cos \alpha \sec \beta \}$, 2α being the vertical angle of the given cone.

4. Show that any section of a cylinder by a plane not parallel or perpendicular to the axis is an ellipse.

If e_1 , e_2 be the eccentricities of two sections at right angles and having a common minor axis, show that

$$e_1^2 + e_2^2 = 1.$$

5. If a right cone be cut by a plane, find under what conditions the section will be a parabola, and give (without proof) a construction for determining the position of its focus.

6. The tangents drawn from any point on the directrix of a conic subtend right angles at the corresponding focus. Prove this, and show that if TP , TQ be the two tangents the conic will be a parabola, ellipse, or hyperbola, according as the angle PTQ is equal to, less or greater than, a right angle. Examine, with particular care, the case of the hyperbola.

7. In an ellipse, if CD be a semi-diameter conjugate to CP , show that

$$CP^2 + CD^2 = AC^2 + BC^2.$$

8. Investigate an expression for the area of a triangle in terms of the coordinates of its three angular points.

E, F, G, H are points in the four sides of a square $ABCD$, such that $AE = \frac{1}{2}AB$, $BF = \frac{1}{3}BC$, $CG = \frac{1}{4}CD$, $DH = \frac{1}{5}DA$. Show that the area of the quadrilateral $EFGH$ is equal to that of a square whose side = $\frac{3}{4}$ of the side of the given square.

9. Find the angle between the two straight lines whose equations are

$$y = m_1x + c_1$$

and

$$y = m_2x + c_2.$$

Show that $(2, -1)$ $(0, 2)$ $(3, 0)$ $(-1, 1)$ are the coordinates of the angular points of a parallelogram and find the angle between its diagonals.

10. Find the equation of a parabola in the form $y^2 = 4ax$, the vertex of the curve being the origin.

If PQ be a double ordinate of the parabola, find the locus of its points of trisection.

11. Investigate the equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, at the point whose coordinates are x', y' .

If CP, CD be conjugate semi-diameters, and $CP = \sqrt{ab}$, show that the normal at D touches the circle whose equation is

$$x^2 + y^2 = (a - b)^2.$$

12. What loci are represented by the equations

$$(i.) \quad x\{(2x + y)^2 - (x + a)(x - a)\} = 0;$$

$$(ii.) \quad x^2 + y^2 + \frac{1}{6}y(x - 1) - 2x + 1 = 0;$$

$$(iii.) \quad x = a\theta + b\theta^2, \\ y = b\theta + a\theta^2;$$

where θ is a variable parameter?

VI. PURE MATHEMATICS (2).

(Further Questions and Problems on the Subjects of the Qualifying Examination, and Theory of Equations.)

[Full marks may be obtained for about four-fifths of this paper. Great importance is attached to accuracy.]

1. Solve the equations

$$\left. \begin{aligned} ax + cy + bz &= a \\ cx + by + az &= b \\ bx + ay + cz &= c \end{aligned} \right\}$$

and find x, y, z in their simplest forms.

2. Sum the series

(i.) $1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \dots$ to n terms.

(ii.) $\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots$ to infinity.

3. Write down an expression for e^x arranged according to ascending powers of x , and thence deduce a series for $\log(1+x)$.

Obtain the first four terms in the expansion of $\log\left(1 - \frac{2x^2}{1+x}\right)$, according to ascending powers of x .

4. State the law for the formation of the successive convergents to a continued fraction.

Express the continued fraction $2 + \frac{1}{3} + \frac{1}{4} + \frac{1}{3} + \frac{1}{4} + \dots$ as a quadratic surd.

5. Prove that the arithmetic mean of n positive quantities is greater than their geometric mean.

Show that $(bc + ca + ab)^2$ is not less than $3abc(a + b + c)$.

6. Find the number of permutations of n things taken r at a time.

How many different words of two consonants and one vowel can be formed from the letters in the word *Woolwich*?

7. Show that the equations

$$x^3 + x^2 + x = -6 \text{ and } x^4 - x^3 + x^2 + 2x = 6$$

have two imaginary roots in common, and solve the equations.

8. Solve the equations

$$(i.) \quad x(x^2 - 4) = a(a^2 - 4).$$

$$(ii.) \quad x^4 + x^2 + 1 = \frac{3}{2}x(x^2 + 1).$$

9. Show that the limit of $\frac{\sin \theta}{\theta}$ when θ is indefinitely diminished is unity.

Find the angles of an isosceles triangle whose equal sides are a and whose base $= a(1 + \lambda)$ where λ is a very small fraction.

If $\lambda = .022$, show without tables that the base angles $= 59^\circ.16'$ nearly.

10. Investigate an expression for the radius of a circle which touches one side of a triangle and the other two sides produced.

If r_1, r_2, r_3 be the radii of the three escribed circles, and

$$\left(1 - \frac{r_1}{r_2}\right)\left(1 - \frac{r_1}{r_3}\right) = 2.$$

show that the triangle must be right-angled.

11. Prove Demoivre's Theorem in the case of a positive index, and

$$2 \cos \theta = x + \frac{1}{x}, \text{ show that } 2 \cos n\theta = x^n + \frac{1}{x^n}.$$

12. The altitudes of a tower are observed at two points A, B , 120 feet apart, situated in a line with the foot of the tower, to be $\theta, 2\theta$, and at a point midway between them to be 45° . Show that the height of the tower $= 109.37$ feet nearly.

VII. PURE MATHEMATICS (3).

(Differential and Integral Calculus.)

[Full marks may be obtained for about four-fifths of this paper. Great importance is attached to accuracy.]

1. Differentiate the expressions

$$\tan^{-1} \frac{x}{\sqrt{1-x^2}}, \quad \log(x + \sqrt{x^2 - a^2}) + \sec^{-1} \frac{x}{a}.$$

2. If $\sin^{-1}y = m \sin^{-1}x$, prove that

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2y = 0.$$

3. Expand $\tan^{-1}x$ in a series of ascending powers of x , and show that the expansion is inapplicable when $x > 1$. In this latter case show how to expand the function in an infinite series.

4. Explain the method of determining the maximum or minimum values of a function of a single variable; and find the maximum or minimum values of

$$\frac{2x+9}{x^2+4x+10}, \text{ and of } \frac{\log x}{x}.$$

5. Prove that, in the parabola, $y^2 = 4mx$, the length of the subnormal is constant; and in the logarithmic curve, $y = a^x$, the subtangent is constant.

6. Find the equation of the tangent at any point on the curve $x^m y^n = a^{m+n}$; and show that the portion of the tangent intercepted by the coordinate axes is divided in a constant ratio at the point of contact.

7. Find the length of the radius of curvature at any point on an ellipse.

8. If a curve be referred to polar coordinates, prove that at a point of inflexion we have

$$\frac{d^2 \left(\frac{1}{r} \right)}{d\theta^2} + \frac{1}{r} = 0.$$

9. Find the following integrals :

$$\int \frac{dx}{\sqrt{1+x-x^2}}, \quad \int \sin^3 x dx, \quad \int x \log x dx.$$

10. Evaluate the definite integrals :

$$\int_0^{\frac{\pi}{2}} \sin^2 x dx, \quad \int_0^1 \frac{dx}{\sqrt{x-x^2}}, \quad \int_0^{\pi} \frac{dx}{5+4 \cos x}.$$

11. Find the whole area of *either* of the curves :

$$(1) \quad a^2 y^2 = x^3 (2a - x).$$

$$(2) \quad \left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1.$$

12. Determine the length of any arc of a parabola measured from its vertex.

VIII. STATICS.

1. Assuming that the resultant of two equal forces bisects the angle between them, prove that the resultant of two forces, one of which is double the other, acts in the diagonal of the parallelogram of which they are sides.

2. A straight uniform lever, weighing 10 lbs., rests on a fulcrum one-third of its length from one end ; it is loaded with a weight of 4 lbs. at that end ; find what vertical force must act at the other end to keep the lever at rest.

3. Four forces act along the sides of the rectangle $ABCD$, and are measured by those sides ; the first three, AB , BC , CD , act in a contrary sense to the fourth, AD . Find their resultant and its line of action.

4. Three forces, PA , PB , PC , diverge from the point P ; and three others, AQ , BQ , CQ , converge to the point Q . Show that the resultant of the six is represented in magnitude and direction by $3PQ$, and passes through the centre of gravity of the triangle ABC .

5. It is required to decompose a force whose magnitude and line of action are given into two equal forces passing through two given points. Give a geometrical construction for solving the problem (1) when the two points are on the same side of the line; (2) on opposite sides.

6. Two uniform rods, AB , BC , jointed together at B , are placed in a horizontal line, ABC , and supported by two props, one at A , the other between B and C . Determine the position of the latter, given the lengths and weights of the two rods.

7. A weight is supported on a smooth inclined plane by a force acting along the plane. Find the force, also the pressure on the plane.

If the plane be rough, prove that this pressure is unaltered; and explain how far the force may be determined.

8. Three given weights are rigidly connected together; prove that the resultant weight always passes through one point, fixed as regards the weights, however the system be displaced in the plane of the weights. Does this hold if the system be displaced out of its original plane?

9. Two weights, joined by a string passing over a pulley, rest on two smooth inclined planes of the same height, and whose highest points coincide at the pulley. Apply the principle of *virtual velocities* to show that the weights are as the lengths of the planes, and that, however they are displaced, their centre of gravity moves horizontally.

Determine by the same principle whether these theorems hold when the pulley is not at the vertex of the planes.

10. Four weightless rods are freely jointed together, forming a quadrilateral $ABCD$. If two equal and opposite forces are applied at two points, one on the rod AB , one on the opposite rod CD , determine the conditions that they shall balance each other.

If, instead of forces, two equal and opposite *couples* be applied to the rods AB , CD , what is necessary for equilibrium?

State generally in each case what will happen if the conditions are not satisfied.

IX. DYNAMICS.

[The measure of the acceleration of gravity may be taken to be 32 when a foot and a second are units of length and time. Great importance will be attached to accuracy.]

1. Define uniform velocity and uniform acceleration, and state how each is measured.

2. A body whose weight is W lies on a smooth table. What horizontal force applied to it will cause it to move with unit acceleration, a foot and a second being the units of space and time?

3. From a train moving with velocity V , a carriage on a road parallel to the line, at a distance d from it, is observed to move so as to appear always in a line with a more distant fixed object, whose least distance from the railway is D . Find the velocity of the carriage.

4. A particle moving from rest with (unknown) uniform acceleration acquires a velocity v in the time t ; find the space described.

5. A particle is projected vertically upwards with a velocity u ; find the height it will reach.

Show that the velocities with which it passes through any assigned point, in its ascent and fall, are equal.

6. Find the conditions that two given non-elastic balls, on striking each other, shall both be reduced to rest. If they are perfectly elastic, what will happen after the collision?

Show this directly, without employing general formulas.

7. State (and prove) with what acceleration a particle will fall down a smooth inclined plane. State also (without proof) the acceleration, if the plane be rough: given μ the coefficient of friction.

A heavy slab whose under surface is rough, but the upper smooth, slides down a given inclined plane. Find the acceleration with which a small particle laid on its upper surface will move along the slab.

8. If a train ascends a gradient of 1 in 40 by its own momentum for a distance of 1 mile, the resistance from friction, etc., being 10 lbs. per ton, find its initial velocity.

9. Find the greatest range which a projectile with an initial velocity of 1,600 ft. per second can attain on a horizontal plane.

Show also that for a small difference of elevation, not exceeding $10'$, whether of excess or defect, the range attained will fall short of the maximum by less than $1\frac{1}{2}$ feet.

10. A body of mass m is placed at a distance R from a centre of force O , which attracts it with a force P . Find the velocity and direction with which it must be started so as to describe a circle round O .

If the force of attraction, for different distances, varies inversely as the squares of the distances, and for different bodies, directly as their masses, prove that if several bodies move round O in concentric circles, the squares of the times of revolution are as the cubes of the radii,

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1888.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.]

1. Define a *right angle*, and a *rhombus*.

If, at a point in a straight line, two straight lines on opposite sides of it make the adjacent angles together equal to two right angles, they are in the same straight line.

AOB , COD are two intersecting straight lines, and each of the figures $AOCE$, $BODF$ is a rhombus. Show that the straight line EF passes through O , and that AC , BD are parallel.

2. The opposite sides and angles of a parallelogram are equal, and either diagonal bisects it.

Straight lines drawn through A , C , the extremities of one diagonal of a parallelogram $ABCD$, respectively perpendicular and parallel to the other diagonal, intersect in E . Prove that $BE = CD$.

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3. If a straight line be divided into any two parts, the rectangle contained by the whole and one of the parts is equal to the rectangle contained by the two parts, together with the square on the aforesaid part.

The diagonal AC of a square $ABCD$ is produced to E , so that $CE = BC$. Prove that the square on BE is equal to the rectangle under AC, AE .

4. Describe a square equal to a given rectangle.

5. Explain fully how you establish the truth of the proposition that "the straight line drawn at right angles to a diameter of a circle at its extremity touches the circle."

Describe a circle passing through a given point, and also touching a given circle in a given point.

6. The angle in a semicircle is a right angle; the angle in a segment greater than a semicircle is acute; the angle in a segment less than a semicircle is obtuse.

The circle described with centre A and radius AB , cuts the circle circumscribing the rectangle $ABCD$ in the point E . Show that CE is equal to AD , and that DE is parallel to AC .

7. From a given circle, cut off a segment which shall contain an angle equal to a given angle.

8. Describe a circle about a given equilateral and equiangular pentagon.

9. If an angle of a triangle be bisected by a straight line which cuts the base, the segments of the base have the same ratio which the sides of the triangle have to one another.

Prove that the radius of the circle inscribed in any isosceles triangle has the same ratio to the perpendicular altitude as the base of the triangle has to its perimeter.

10. On a given straight line construct a quadrilateral figure similar and similarly described to a given quadrilateral figure.

11. One of the sides containing the right angle of a triangle is double of the other, and circles are described having these two sides as diameters. Show that the length of their common chord is two-fifths of the hypotenuse of the triangle.

12. In equal circles, angles, whether at the centres or at the circumferences, have the same ratio to one another as the arcs on which they stand.

II. ALGEBRA.

(Up to and including the Binomial Theorem; the theory and use of Logarithms.)

1. Simplify the expression

$$\frac{\{(a+b)(a+b+c)+c^2\}\{(a+b)^2-c^2\}}{\{(a+b)^3-c^3\}\{(a+b+c)\}}.$$

2. Show that $x^3+y^3+z^3-3xyz$ is divisible exactly by $x+y+z$; and hence, or otherwise, show that

$$(b-c)^3+(c-a)^3+(a-b)^3=3(b-c)(c-a)(a-b).$$

3. Find the highest common factor of $16x^4+36x^2+81$ and $8x^3+27$; and write down the least common multiple of $8x^3+27$, $16x^4+36x^2+81$, and $6x^2-5x-6$.

4. Extract the square root of

$$4x^2a^{-2}+16x^{-2}a^2-12\frac{x}{a}-24\frac{a}{x}+25.$$

5. Determine what relation must hold between a , b , and c in order that the roots of the equation $ax^2+bx+c=0$ may be real and different.

If a , b , c , x are all real quantities in the equation

$$(a^2+b^2)x^2-2b(a+c)x+b^2+c^2=0,$$

prove that a , b , c are in G.P., and that x is their common ratio.

6. Solve the equations

$$(1) \quad \frac{1}{2}(x-2) - \frac{x-5}{9} + 5\frac{x-1}{6} = 0;$$

$$(2) \quad \frac{b}{x-a} + \frac{a}{x-b} - 2 = 0;$$

$$(3) \quad \left. \begin{array}{l} x+y+z=19 \\ x^2+y^2+z^2=133 \end{array} \right\} \text{having given that } y \text{ is a mean pro-} \\ \text{portional to } x \text{ and } z.$$

7. Two casks, A and B , are filled with two kinds of sherry, mixed in cask A in the ratio $2:7$, and in cask B in the ratio $1:5$; what quantity must be taken from each to form a mixture which shall consist of 2 galls. of the first kind and 9 galls. of the second?

8. Prove the formula for the sum of n terms of an Arithmetic Progression whose first and last terms are given.

If s_1, s_2, s_3 , etc., are the sums of m Arithmetic series, each to n terms, the first terms being 1, 2, 3, etc., respectively, and the differences 1, 3, 5, etc., respectively, show that

$$s_1 + s_2 + s_3 + \dots s_m = \frac{1}{2}mn(mn + 1).$$

9. The number of combinations of n letters, taken 5 together, in which a, b, c occur is 21. Find the number of combinations of them, taken 6 together, in which a, b, c, d occur.

10. Write down the two middle terms in the expansion of

$$\left(x + \frac{1}{x}\right)^{2m+1};$$

and show that, if x be a small fraction,

$$\frac{\sqrt{1+x} + \sqrt[3]{(1-x)^2}}{1+x + \sqrt{1+x}}$$

is very nearly equal to $1 - \frac{5}{8}x$.

11. Prove that the logarithm of the quotient of two numbers is the difference of the logarithms of the numbers.

If the logarithms of a, b, c be respectively p, q, r , prove that $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$.

Prove that $\log \frac{7}{18} - 2 \log \frac{5}{8} + \log \frac{32}{144} = \log 2$.

12. If the number of persons born in any year equals $\frac{1}{46}$ th of the whole population at the beginning of the year, and the number who die equals $\frac{1}{66}$ th of it, find in how many years the population will be doubled. [Use the logarithm tables supplied.]

III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy. Questions 1, 8, 9, 10, 11 may be worked with the aid of the logarithm tables supplied.]

1. Prove that, to turn circular measure into seconds, we must multiply by 206265; and, to turn seconds into circular measure, we must multiply by '000004848, approximately. [$\pi = 3.14159265\dots$]

Determine the circumference and the diameter of the Earth in geographical miles [60 to a degree of latitude], each degree subtending 1° at the centre of the Earth.

2. Define the *sine*, *cosine*, *tangent*, *cotangent*, *secant*, *cosecant*, and *versed sine* of an angle, illustrating by reference to a figure; and express them all in terms of the secant.

Construct a table showing their values for angles of 0° , 30° , 45° , 60° , 90° , 120° , 180° .

3. Prove that

$$(i.) \quad \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \text{ (by geometrical construction);}$$

$$(ii.) \quad \tan \alpha - \tan \beta = \sin(\alpha - \beta) \sec \alpha \sec \beta;$$

$$(iii.) \quad \sec \theta + \tan \theta = \sqrt{\left(\frac{1 + \sin \theta}{1 - \sin \theta}\right)} = \tan\left(\frac{1}{4}\pi + \frac{1}{2}\theta\right).$$

4. Determine, in circular measure, all the angles given by the equation

$$\cos \theta - \sin \theta = \sqrt{2}.$$

Prove that, if $\cos \theta = \tan \theta$,

$$\text{then} \quad \sin \theta = 2 \sin \frac{\pi}{10}.$$

5. Construct a table expressing the *inverse* trigonometrical functions in terms of each other.

Prove that

$$\begin{aligned} \cos^{-1} \sqrt{\frac{a-x}{a-b}} &= \sin^{-1} \sqrt{\frac{x-b}{a-b}} = \cot^{-1} \sqrt{\frac{a-x}{x-b}} \\ &= \frac{1}{2} \sin^{-1} \frac{\sqrt{(a-x)(x-b)}}{\frac{1}{2}(a-b)}. \end{aligned}$$

6. Prove that, if

$$\cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha,$$

then

$$\frac{x^2}{a^2} - 2 \frac{xy}{ab} \cos \alpha + \frac{y^2}{b^2} = (\sin \alpha)^2.$$

7. Prove that, in every triangle, $a \operatorname{cosec} A = b \operatorname{cosec} B = c \operatorname{cosec} C =$ the diameter of the circle circumscribing the triangle.

In determining the distance of a far object C , a base AB is measured, and the angles ABC , BAC are observed, each angle being very nearly a right angle. Prove that the distance may be taken to be

$$AB \operatorname{cosec} (180^\circ - ABC - BAC).$$

8. Given the sides a , b , c of a triangle, write down expressions for determining the angles and the area of the triangle, and the radii of the circles touching its sides.

Given, in feet, $a = 10$, $b = 24$, $c = 26$, determine the angles, in degrees and minutes, and the area of the triangle, in square feet.

9. From a boat the angles of elevation of the highest and lowest points of a flagstaff, 30 feet high, on the edge of a cliff are observed to be $46^\circ 12'$ and $44^\circ 13'$; determine the height of the cliff and its distance.

10. Determine the diameter of a cylindrical gasholder to contain 10 million cubic feet of gas, supposing the height to be made equal to the diameter; and determine in tons the weight of iron plate, weighing $2\frac{1}{2}$ pounds per square foot, required in the construction of the gasholder, supposing it open at the bottom, and closed by a flat top.

11. Determine the number of cubic yards in a bank of earth on a horizontal rectangular base 60 feet long and 20 feet broad, the four sides of the bank sloping up to a ridge at an angle of 40° to the horizon.

IV. STATICS AND DYNAMICS.

1. Show how to place three forces, represented by the numbers 3, 4, 5, at a point so as to balance each other.

Would it be possible for the forces 2, 3, 5 to make equilibrium?

2. Three equal forces, P , diverge from a point, the middle one being inclined at an angle of 60° to each of the others. Find the resultant of the three.

3. A uniform lever, weighing 30 lbs. and 16 feet long, is loaded at either end with weights of 20 and 30 lbs. respectively. Find where the fulcrum must be placed for equilibrium.

4. Prove that the moments of any two non-parallel forces, about any point on their resultant, are equal and opposite.

If two forces form a couple, show that the algebraical sum of their moments about any point in the plane is constant.

5. Show how a weight of 2 lbs. can be balanced by a weight of 1 lb., by means of pulleys; and show that the former will rise through half the space which the latter descends.

6. State, and prove, what power acting along a smooth inclined plane is required to support a weight resting on the plane.

A heavy string is placed with part of it resting on a given inclined plane, the remaining part hanging vertically over a small pulley at the top of the plane. Find the point of the string which must be placed over the pulley, for equilibrium.

7. Explain, and prove, the Parallelogram of Velocities.

Decompose a given velocity, v , into two components whose directions are each inclined at 30° to its own.

8. What is meant in Dynamics by the *Mass* of a body? What result of observation proves that the Weights of all bodies are proportional to their Masses?

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9. Explain clearly what is meant by saying that $g = 32$, where g is the acceleration due to gravity.

If the unit of time were altered to 2 sec., what would be the value of g ; and why?

10. In what time will a body fall from rest through 100 ft.? If it be retarded in its fall by the tension of a string attached to it, so as to occupy 5 sec. in the fall, what is the tension of the string, the weight being given?

11. A body falls freely through 100 ft. from rest, with what velocity will it reach the ground?

If, instead of falling from rest, it be projected downwards so as to reach the ground with twice the former velocity, find the initial velocity.

12. A ball is rolled across a smooth table, and leaves it with a given horizontal velocity. What curve does it describe during its fall?

If v be the initial velocity, and h the height of the table, find the horizontal distance from the table of the point where the ball reaches the floor.

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[Full marks may be obtained for about four-fifths of this paper. Great importance is attached to accuracy.]

1. Show that, if from the focus of a parabola a perpendicular be drawn to any tangent, the point of intersection lies on the tangent at the vertex.

Having given the extremities of a focal chord of a parabola, and also the focus find the position of the vertex.

2. Prove that the area of the parallelogram of which two semi-diameters of an ellipse are adjacent sides is invariable, if the diameters be conjugate.

Show that the sum of the lengths of two conjugate diameters of an ellipse is never less than the sum of the lengths of the principal axes.

3. Define a hyperbola by means of the relation between the distances of any point upon it from a given point and a given line, and deduce from the definition the property that the difference between the focal distances of a point on the curve is invariable.

Two circles in the same plane are external to each other. Show that the locus of the centre of a circle touching them externally is a hyperbola, and determine the position of the asymptotes.

4. Prove that the focal radii of a point of a central conic section make equal angles with the tangent at that point.

Show that an ellipse and a hyperbola which have the same foci cut one another at right angles.

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5. Establish the focus and directrix property of the section by a plane of a right circular cone.

Determine the eccentricity (1) of a circle; (2) of a pair of parallel straight lines.

6. Find the rectangular coordinates of a point dividing in a given ratio the straight line joining two points with given coordinates.

Examine the case when the given ratio = -1.

7. The equations to two straight lines being given, find the equation to a straight line joining a given point to the point of intersection of the straight lines.

Find the equation to the straight line passing through the origin and the intersection of the straight lines given by

$$\frac{x}{a} + \frac{y}{b} = 1, \quad \frac{x}{b} + \frac{y}{a} = 1.$$

8. Show how to determine the points where the straight line represented by

$$y = mx + b$$

cuts the locus of the equation

$$(x - \alpha)^2 + (y - \beta)^2 = k^2.$$

When will the straight line touch the locus?

9. Find the equation to the normal to the parabola given by $y^2 = 4ax$ at the point whose coordinates are (x', y') . Show that not more than three normals can be drawn to a parabola from a point in its plane.

10. Show how to determine the pair of diameters common to the circle given by $x^2 + y^2 = k^2$ and the locus of $ax^2 + by^2 + 2hxy + c = 0$. Under what conditions will the diameters become coincident?

11. Determine the locus of the intersection of the straight lines given by

$$\frac{tx}{a} - \frac{y}{b} + t = 0, \quad \frac{x}{a} + \frac{ty}{b} - 1 = 0,$$

t being a variable parameter.

What connection is there between the angle $2 \tan^{-1} t$ and the point on the locus where the two straight lines intersect?

12. Discuss and trace the loci of the following equations :—

(1) $(3x - y)^2 = 6x - 2y$;

(2) $\frac{a}{x} + \frac{b}{y} = 1$;

(3) $x = a \cos(\theta + \alpha)$, $y = b \cos(\theta + \beta)$, where θ is a variable parameter.

VI. PURE MATHEMATICS (2).

[Full marks may be obtained for about four-fifths of this paper. Great importance is attached to accuracy.]

1. Find x in its simplest form from the equation

$$x^3 - 3x^2 + 1 = 0.$$

2. Prove that the greatest and least values of

$$\frac{6x^3 - 22x + 21}{5x^2 - 18x + 17}$$

are $\frac{5}{4}$ and 1, corresponding to the values 1 and 2 of x .

3. Find x , y , and z from the equations

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1,$$

$$\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 1,$$

$$\frac{x}{4} + \frac{y}{5} + \frac{z}{6} = 1.$$

Exhibit the solution also by means of determinants.

4. Prove that, if A varies as B^m when C is constant, and if A varies as C^n when B is constant, then will A vary as $B^m C^n$ when both B and C vary. Determine the resistance of the air to a projectile 16 inches in

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diameter at a velocity of 1500 feet per second, supposing the resistance of the air varies as the square of the diameter and the cube of the velocity; given that the resistance of the air to a projectile 1 inch in diameter at a velocity of 1000 feet per second is $2\frac{1}{2}$ pounds' weight.

5. Prove that the equations

$$x + y + z = 100,$$

$$5x + y + \frac{z}{20} = 100,$$

have only one solution in positive integers; and determine this solution.

A man bought 100 animals, consisting of oxen, sheep, and rabbits, for £100; the oxen costing £5, the sheep £1, and the rabbits 1s. each; how many of each did he buy?

6. Resolve into partial fractions the expression

$$\frac{x^4}{(x-1)(x-2)(x-3)};$$

and then expand the fractions in positive powers of x , writing down the coefficient of x^n .

7. Explain how the theorems

$$(i.) \quad a^x \times a^y = a^{x+y},$$

$$(ii.) \quad a^x \div a^y = a^{x-y},$$

$$(iii.) \quad (a^x)^r = a^{xr},$$

$$(iv.) \quad \sqrt[r]{a^x} = a^{x \div r},$$

are established for *all* values of x , y , and r .

Examine the truth of the following:—

$$(v.) \quad e^{\frac{1}{r}} = \sqrt[r]{e} = e,$$

$$(vi.) \quad \log [\log \{\log e^{e^e}\}] = 1,$$

the logarithms being to the base e .

8. Determine x in degrees and minutes from the equation

$$6 \cos x + 8 \sin x = 9.$$

Give also a graphical method of solution, the sketch being made freehand.

9. Prove that

$$\theta - n\pi = \tan \theta - \frac{1}{3}(\tan \theta)^3 + \frac{1}{5}(\tan \theta)^5 - \dots;$$

and determine the value which must be given to n when θ lies between

$$\frac{11\pi}{4} \quad \text{and} \quad \frac{13\pi}{4}.$$

10. Prove that, when resolved into real and imaginary parts,

$$\cos(u + iv) = \cos u \cosh v - i \sin u \sinh v,$$

where $\cosh v$ denotes $\frac{1}{2}(e^v + e^{-v})$, $\sinh v$ denotes $\frac{1}{2}(e^v - e^{-v})$, and $i = \sqrt{-1}$.

Prove also that

$$\cos iv = \cosh v, \quad \sin iv = i \sinh v.$$

11. Prove the trigonometrical summations

$$\sum_{r=0}^{r=n} \cos ra = \cos \frac{1}{2}na \sin \frac{1}{2}(n+1)a \operatorname{cosec} \frac{1}{2}a;$$

$$\sum_{r=0}^{r=n} \sin ra = \sin \frac{1}{2}na \sin \frac{1}{2}(n+1)a \operatorname{cosec} \frac{1}{2}a.$$

VII. MECHANICS.

[*Mathematical instruments must be used for questions requiring a graphical method of solution.*]

1. Find the locus of all points in a plane, such that two forces given in magnitude and position shall have equal moments, and in the same sense round any one of the points.

2. AE, DF are vertical walls, A and D being in the same horizontal line. A string $ABCD$ supports two weights at B and C , the weight at B being 1 lb. Find to two places of decimals the weight at C and the tensions of the three strings

$$\angle AB = 40^\circ, \quad \angle ABC = 150^\circ, \quad \text{and} \quad \angle BCD = 90^\circ.$$

[This problem had to be solved by graphical construction on a lithographed plan.]

3. Three forces, 5.05 lbs., 4.24 lbs., 3.85 lbs. in magnitude, act at three given points of a flat disk resting on a smooth table. Place the forces, by geometrical construction, so as to keep the disk in equilibrium; and measure and write down the number of degrees contained in each of the three angles they make with one another.

4. A vertical beam, of weight W , is constrained by guides so as to move only in its own direction, the lower end resting on a smooth floor. If a smooth inclined plane of given slope be pushed under it by a horizontal force acting at the back of the inclined plane, so as to raise the beam, find the force required.

If there is friction between the floor and the inclined plane, but none elsewhere, what must be the least value of μ , if the inclined plane will remain, when left in a given position under the beam, without being forced out; given the weight of the inclined plane?

When there is no friction, verify in this case that what is gained in power is lost in velocity.

5. State the analytical conditions necessary for the equilibrium of any number of forces in a plane, given the forces and the coordinates of their points of application.

Show that these equations are unaltered if the point of application of any force be changed to any other point on its line of action.

6. A particle is projected along a rough horizontal plane with a given velocity; what is its acceleration? If at each point P of its path a vertical PH be drawn, such that its velocity at P is that due to a fall through the height PH , find the locus of H .

Hence, deduce a simple geometrical construction for determining the point at which it will stop.

7. A weight P is descending, drawing upwards a smaller weight Q by means of a string passing over a smooth pulley; if, when they have each moved through a space h from rest, the string is suddenly cut, find by means of the principle of Work, or otherwise, the additional height to which Q will ascend.

8. How is *angular velocity* usually measured? Find the angular velocity of the Earth round its axis.

If a line turns round its extremity O , which is a fixed point within a given circle, with uniform angular velocity, show that the velocity with

which its intersection P with the circle travels along the circumference varies as PH , which is a line drawn from P through the centre, to meet OH , a perpendicular to OP .

9. Find the equation of the path of a projectile, referred to horizontal and vertical axes.

Show that all projectiles the horizontal component of whose initial velocities is the same describe equal parabolas.

Prove that it is impossible for two projectiles discharged together from the same point to meet each other in their flight.

10. A sphere at rest is struck by an equal sphere moving with given velocity in a line inclined at 30° to the line joining the centres at the instant of contact. Find the motions of both after impact; the elasticity being perfect.

Determine also the motions of their common centre of gravity before and after impact.

11. An elastic string is gradually stretched by an increasing force. When the string has been lengthened 2 inches, the force is found to amount to 10 lbs. What is the work in foot-pounds done by the force, assuming that the force is always proportional to the elongation produced?

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1889.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.]

1. At a given point in a given straight line make a rectilineal angle equal to a given rectilineal angle.

Having given one *side* of a right-angled triangle and the sum of the other side and the hypotenuse, construct the triangle.

2. Write out Euclid's definition of, and axiom on, *parallel straight lines*.

Enunciate the first proposition proved by means of the axiom.

3. If a straight line AB be divided internally in C , how can the rectangle $AC \cdot CB$ be expressed in terms of the squares on AB , BC , CA ?

If four equal circles are described round a square so that each circle touches externally a side of the square and the two adjacent circles, show that the rectangle contained by twice the diameter of one of the circles and a side of the square is equal to the difference between the given square and the square on the diameter of one of the circles.

4. In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle will be greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

5. The opposite angles of a quadrilateral inscribed in a circle are together equal to two right angles.

If a quadrilateral have one vertex at the centre of a circle and the other three on the circumference, show that of the angles at these three vertices one will be equal to the sum of the other two.

6. A segment of a circle being given, describe the circle of which it is a segment.

7. Define *pentagon*, *hexagon*, *quindecagon*.

Inscribe an equilateral and equiangular pentagon in a given circle.

8. If two triangles have one angle of the one equal to one angle of the other, and the sides about the equal angles proportionals, the triangles shall be equiangular to one another, and shall have those angles equal which are opposite to the homologous sides.

If D and E are points on the base BC of a triangle ABC such that AB , AC are respectively mean proportionals between BC and BD , BC and CE ; show that AD and AE are each a mean proportional between BD and CE .

9. If the diameter of the circumscribing circle of an isosceles triangle is n times each of the equal sides, show that each of these equal sides is n times the perpendicular from the vertex of the triangle to the base.

10. Similar triangles are to one another in the duplicate ratio of their homologous sides.

If AB , CD be two chords of a circle intersecting in an external point E , show that the triangle EAC has to the triangle EBD the duplicate ratio of that which AC has to BD .

11. If two similar parallelograms have a common angle and be similarly situated, they are about the same diameter.

12. Chords are drawn through a point A on the circumference of a circle whose centre is O . Show that the locus of the middle points of these chords is a circle whose diameter is AO .

II. ALGEBRA.

(Up to and including the Binomial Theorem; the theory and use of Logarithms.)

[N.B.—Great importance will be attached to accuracy.]

1. Resolve into simple factors

$$\{2ax + (a-c)y - 2cz\}^2 - \{2cx + (c-a)y - 2az\}^2,$$

and show that $a-c$ and $x+z$ are factors of

$$(ax + by - cz)^2 - (cx + by - az)^2.$$

2. Find the highest common factor of

$$3a^3 + 14a^2 + 12a + 16 \text{ and } 2a^4 + 7a^3 - 4a^2 - a - 4.$$

3. Find the sum of

$$\frac{1}{3x^2 - 14xy + 15y^2} \text{ and } \frac{2}{3x^2 - 2xy - 5y^2}$$

and divide it by the difference between

$$\frac{x+y}{x-3y} \text{ and } \frac{x-y}{x+3y}.$$

4. A merchant gained as many sovereigns on a certain quantity of coal as there were shillings in the cost price of a ton, or pence in the retail price of a cwt. How many tons did he sell?

5. Simplify

$$\sqrt{\frac{\left(\frac{1}{a} + \frac{1}{b}\right)^2 - \frac{1}{ab} - \frac{4}{(a-b)^2}}{\left(\frac{1}{a} - \frac{1}{b}\right)^2 - \frac{1}{ab}}},$$

and find the value of $27x + 24x^2 - x^3$, when $2x = \sqrt{21} - 3$.

6. Solve the equations

$$(1) \frac{x - \frac{1}{2}}{3} - \frac{\frac{x}{3} - 12}{5} = \frac{2x - 1}{3} + \frac{9\frac{1}{2} - x}{21}.$$

$$(2) \begin{cases} x + 2y + 3z = 4, \\ x + 3y + 2z = 4z, \\ x + 2z + 3 = 4y. \end{cases}$$

$$(3) \frac{x-3}{x-2} + \frac{x+3}{x+2} = \frac{2x+3}{x+1}.$$

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7. Find (i.) the sum, (ii.) the sum of the squares, (iii.) the sum of the cubes, of the roots of the equation

$$x^2 - px + q = 0.$$

8. A bag contains some sovereigns, three times as many half-sovereigns, and also some florins and half-crowns. The value of the gold coins is double that of the silver coins, and the half-crowns are worth half as much again as the florins. Compare the ratios of the numbers of the several coins.

9. Find the sum of 31 terms of the series

$$4\frac{1}{2} + 4\frac{1}{8} + 3\frac{9}{16} + \text{etc.}$$

If
$$\frac{1}{a} + \frac{1}{c} = \frac{1}{2b-a} + \frac{1}{2b-c},$$

prove that $2b$ is either the arithmetic mean between $2a$ and $2c$, or the harmonic mean between a and c .

10. If ${}^nC_r$ denote the number of combinations of n things taken r together, show by general reasoning that ${}^nC_r = {}^nC_{n-r}$.

Also show that

$${}^{n+2}C_{r+1} = {}^nC_{r+1} + 2 \times {}^nC_r + {}^nC_{r-1}.$$

11. Find the first five terms in the expansion of $(1-3x)^{-\frac{2}{3}}$ by the Binomial Theorem; and, if $(1+x+x^2+x^3)^{-4}$ be expanded in a series of powers of x , prove that the coefficient of x^8 will be 14.

12. Write down the values of $\log_{10} .004$ and $\log_4 100$.

Also determine how many terms of the series

$$.04, .08, .16, .32 \dots$$

will amount to 41943.

III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy. Questions 1, 8, 9, 10, 11 may be worked with the aid of the logarithm tables supplied.]

1. The value of the divisions on the outer rim of a graduated circle is $5'$, and the distance between two successive divisions is $\cdot 1$ of an inch; find the radius of the circle. [$\pi = 3\cdot 141593$.]

A church spire whose height is known to be 45 feet subtends an angle of $9'$ at the eye; find its distance approximately.

2. Find the smallest angle, positive or negative, which has the same sine and cosine as $\frac{3\pi}{2} + \alpha$. $\left[\alpha < \frac{\pi}{2} \right]$

If $\tan^2 \theta = \frac{5}{4}$, find $\text{versin } \theta$, and explain the double result.

3. Prove that $\cos A - \sin A$ is a factor of $\cos 3A + \sin 3A$; and that

$$\cos^2 A + \cos^2 \left(A + \frac{2\pi}{3} \right) + \cos^2 \left(A - \frac{2\pi}{3} \right) = \frac{3}{2}.$$

Simplify

$$\left\{ \cot \theta + \cot \left(\theta - \frac{\pi}{2} \right) \right\} \left\{ \tan \left(\frac{\pi}{4} - \theta \right) + \tan \left(\frac{\pi}{4} + \theta \right) \right\}.$$

4. Find an expression for all the angles which have the same cosine.

Find all the values of x which satisfy the equation

$$\tan 2x = \tan \frac{2}{x}.$$

5. Express $\sin A \pm \cos A$ in terms of $\sin 2A$, and hence, if $2A$ is in the fourth quadrant, find $\sin A$ in terms of $\sin 2A$.

Show that $2 \sin 11^\circ 15' = \sqrt{2 - \sqrt{2 + \sqrt{2}}}$.

6. Solve the equations

$$(1) \cos nA - \cos(n-2)A = \sin A.$$

$$(2) \cos^{-1} \frac{1-x^2}{1+x^2} + \tan^{-1} \frac{2x}{1-x^2} = \frac{4\pi}{3}.$$

WOOLWICH ENTRANCE EXAMINATION.

7. Prove that in every triangle

$$a = c \cos B + b \cos C,$$

and

$$\sin \frac{B-C}{2} = \frac{b-c}{a} \cos \frac{A}{2}.$$

8. The angular altitude of a lighthouse seen from a point on the shore is $12^{\circ} 31' 46''$, and from a point 500 feet nearer to it is $26^{\circ} 33' 55''$. Required its height above the shore.

9. In the ambiguous case of plane triangles, if C, C', c, c' are corresponding values of an angle and side to be determined,

$$\frac{\sin C}{c} = \frac{\sin C'}{c'}.$$

Prove this by general reasoning, and find the area of the larger of the two triangles determined from the data

$$A = 31^{\circ} 15', \quad a = 5 \text{ inches}, \quad b = 7 \text{ inches}.$$

10. It is proposed to add to a square lawn measuring 58 feet on a side two circular ends, the centre of each circle being the point of intersection of the diagonals of the square. How much turf will be required for the purpose?

11. A hollow pontoon has a cylindrical body 20 feet long, and hemispherical ends, and is made of metal $\frac{1}{8}$ of an inch thick. The outside diameter is 3 ft. 4 in. Find its weight, having given that a cubic inch of the metal weighs 4.5 oz.

IV. STATICS AND DYNAMICS.

(Obligatory Paper.)

1. Enunciate the "Parallelogram of Forces" and the "Triangle of Forces." Three forces in equilibrium are in the ratio $1 : \sqrt{3} : 2$. At what angles do they act?

2. A uniform rod AB , resting with one end A against a smooth vertical wall, is supported by a string BC tied to a point C vertically above A and to the other end B of the rod. Draw a diagram showing the lines of action of the forces which keep the rod in equilibrium, and show that the tension of the string must be greater than the weight of the rod.

3. Find the resultant of two like parallel forces acting on a body.

Two men carry a heavy cask, weight $1\frac{1}{2}$ cwt., which hangs from a light pole, length 6 feet, each end of which rests on a shoulder of one of the men. The point from which the cask is hung is one foot nearer to one point of support than to the other. What is the pressure on each shoulder?

4. Find the relation between the power and the weight in the system of pulleys in which the same string passes round all the pulleys (neglecting the weight of the pulleys).

A 12-stone man raises 3 cwt. by means of such a system, having 4 pulleys in each block and the string attached to the upper block. What will be his pressure on the ground if he pulls vertically downwards?

5. Three equal masses are at the angular points, A, B, C of a triangle. Find their centre of gravity. If the masses are unequal, what must be their ratios in order that their C.G. may be half way between A and the middle point of BC ?

6. What force, acting horizontally, would keep a mass of 16 lbs. at rest on a smooth inclined plane whose height is 3 feet and length of base 4 feet? What would be the pressure on the plane?

7. Define "Velocity," and explain how it is measured numerically (1) when uniform, (2) when variable.

A man walks with velocity 12 along a horizontal platform which is rising vertically with velocity 5. Determine the direction and velocity of his actual motion.

WOOLWICH ENTRANCE EXAMINATION.

8. Explain any apparatus by which the acceleration of gravity may be measured.

9. A stone A is thrown vertically upwards with a velocity of 96 feet per second. How high will it rise?

After 4 seconds from the projection of A another stone B is let fall from the same point. Prove that A will overtake B after 4 seconds more. ($g = 32$.)

10. Enunciate the law of motion which tells us how to measure force. A heavy truck, of mass 16 tons, is standing at rest on a smooth line of rails. A horse now pulls at it steadily in the line of the rails with a force equal to the weight of 1 cwt. How far will he move it in 1 minute? ($g = 32$.)

11. Explain what is meant when it is said that a particle moving with velocity V of uniform magnitude in a circle of radius r has at every instant an acceleration along the radius of the circle.

What is the measure of this acceleration?

12. A stone is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height h . Find where it will strike the level ground through the foot of the tower. What will be its striking velocity?

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

*[Full marks may be obtained for about four-fifths of this paper.
Great importance is attached to accuracy.]*

1. Define a parabola, and prove that the tangent at any point bisects the angle between the focal distance and the perpendicular on the directrix.

A thread has both its extremities fastened at a point A on a sheet of paper, and is kept stretched by two pencil nibs, P and Q , touching the paper. If P and Q move so that AP and AQ are always equal to each other, and PQ always parallel to a fixed line, prove that two equal parabolic arcs will be traced by the pencils upon the paper, and that they will intersect at right angles.

2. Prove that the subnormal at every point of a parabola is equal to the semi-latus rectum.

If M be the foot of the perpendicular from any point P of a parabola upon the tangent at the vertex, prove that the line through M parallel to the tangent at P always touches a parabola whose latus rectum is double that of the original parabola.

3. If the normal at the point P of an ellipse (centre C) meet the axis major in G , and N be the foot of the ordinate of P , prove that CG bears a constant ratio to CN .

If CP produced meet the tangent at one extremity (A) of the major axis in Q , and QE be drawn parallel to the normal at P , meeting CA in E , prove that AE is equal to the semi-latus rectum.

4. Define the auxiliary circle of the ellipse, and prove that the locus of the intersection of the normal to the ellipse at any point (P) with the radius to the corresponding point (Q) of the auxiliary circle is another circle concentric with both curves.

5. Prove that the part of every tangent to an hyperbola intercepted between the asymptotes is bisected at the point of contact.

If the hyperbola be equilateral, prove that the part of every normal intercepted between the major and minor axes is bisected at the point where it meets the curve.

6. Prove that the section of a right cone by a plane, if a closed curve, is an ellipse.

If a number of elliptic sections be made with the major axis the same in length for all, prove that the eccentricity of any section is proportional to the difference of the distances of the extremities of its major axis from the cone's vertex.

7. Prove that $y = mx + c$ is the equation of a straight line, and interpret the constants.

Two straight lines cut the axis of x at distances a and $-a$ and the axis of y at distances b and b' from the origin respectively. Find the co-ordinates of their point of intersection.

8. If in the last case any number of pairs of lines be drawn with a the same for all and $bb' = a^2$, in each pair, prove that the locus of the points of intersection of all the pairs is a circle.

9. Find the equation of the parabola whose vertex and focus are each on the axis of x at distances (a) and (a') from the origin respectively.

Two parabolas have a common axis and concavities in opposite directions; if any line parallel to the common axis meet the parabolas in P and P' , prove that the locus of the middle point of PP' is another parabola, provided the latera recta of the given parabolas are unequal.

10. Find the equation of the chord of contact of tangents drawn from the point h, k , to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

11. Investigate the loci expressed by the equations

$$(1) \quad (y-b)(y-b') + (x-a)(x-a') = 0;$$

$$(2) \quad 2x^2 + 3y^2 - 4ax + 6ay + 4a^2 = 0;$$

$$(3) \quad x = a\left(\mu + \frac{1}{\mu}\right), \quad y = b\left(\mu - \frac{1}{\mu}\right),$$

where μ is variable.

VI. PURE MATHEMATICS (2).

[Full marks may be obtained for about two-thirds of this paper. Great importance is attached to accuracy.]

1. If $x + \frac{1}{x} = y$, express $x^3 + \frac{1}{x^3}$ and $x^5 + \frac{1}{x^5}$ in terms of y .

2. Solve the simultaneous equations

$$x^2 + 2xy + 3y^2 = 43, \quad 2x^2 + 3xy + 5y^2 = 71.$$

3. Show that the expression $-1 + 2\sqrt{-1}$ is a root of the equation

$$x^4 - 12x - 5 = 0;$$

and hence find all its roots.

4. If the expression

$$(ax^2 + 2bx + c + y)^2 - a(ax^4 + 4bx^3 + 6cx^2 + 4dx + e)$$

is a perfect square, show that y must be a root of a certain cubic equation, and find this equation.

5. Resolve $\frac{9x^2 - 7x + 2}{x^4 - x^3 + x - 1}$ into its partial fractions.

6. Find, by aid of the Binomial Theorem or otherwise, the value of the expression

$$\sqrt{x^4 + ax^2 + bx + c} - \sqrt{x^4 + a'x^2 + b'x + c'},$$

when

$$x = \infty.$$

7. Find the values of x which satisfy the equations

$$(i.) \quad \sin 7x = \sin 4x - \sin x ;$$

$$(ii.) \quad \cos 2x + 3 \cos x = 0.$$

8. Find the sum of n terms of the series

$$\sin \frac{\theta}{2} + \sin \frac{3\theta}{2} + \sin \frac{5\theta}{2} + \text{etc.}$$

9. In a plane triangle prove the identical relation

$$\sin 3A \sin(B - C) + \sin 3B \sin(C - A) + \sin 3C \sin(A - B) = 0.$$

10. Prove that $(x + iy)^m + (x' + iy')^n$

can be expressed in the form $R(\cos \phi + i \sin \phi)$,

where $i^2 = -1$, and m and n are any integers. Show how to find the values of R and ϕ .

$$11. \quad \text{If } \frac{x}{e^x - 1} \equiv 1 + B_1 x + \frac{B_2}{1 \cdot 2} x^2 + \frac{B_3}{1 \cdot 2 \cdot 3} x^3 + \frac{B_4}{1 \cdot 2 \cdot 3 \cdot 4} x^4 + \text{etc.};$$

find the values of B_1 , B_2 , B_3 , and B_4 , assuming the ordinary expansion for e^x .

VII. MECHANICS.

[Mathematical instruments must be used for questions requiring a graphical method of solution.]

1. A couple is balanced by two forces in its plane. Prove that they form a couple.

Four forces acting along, and represented by, the sides of a parallelogram form two couples of opposite senses. Prove that they are in equilibrium.

2. Solve graphically the following problem.

A uniform beam APB , weight 100 lbs., is held at any angle of 31° to the vertical by the tension of a string PH , the lower end A resting on a rough horizontal plane.

$$AB = 4\frac{1}{2} \text{ in.}, \quad PB = .8 \text{ in.}, \quad \angle APH = 98^\circ.$$

Find the tension of the string in lbs.; also find (in degrees) the least possible value for the angle of friction ($\tan^{-1}\mu$) at A .

3. A particle rests on a rough horizontal plane: if the plane be gradually raised at one end, find the angle of slope at which the particle will slip.

A cubical block rests on a rough plank with its edges parallel to the edges of the plank. If, as the plank is gradually raised, the block turns over on it before slipping, what must the coefficient of friction (at least) be?

4. A heavy cube (weight 100 lbs.) rests on a rough floor (on which it cannot slip). Prove that the least force required to begin to raise one edge of it off the ground is about $35\frac{1}{2}$ lbs.; find where and in what direction this force must be applied to the block.

Show that the force sufficient to begin to raise it is sufficient, if continued to be applied, to completely overturn it.

5. A true balance has one scale fraudulently loaded. If a body successively weighed in the two scales appears to weigh P and Q pounds, find the amount of the unjust load.

The beam of a balance is 6 ft. long, and it appears correct when empty. A certain body placed in one scale weighs 120 lbs., when placed in the other 121 lbs. Show that the fulcrum must be distant about $\frac{1}{18}$ of an inch from the centre of the beam.

6. Find approximately the number of foot-pounds of work expended in drawing a weight of 100 lbs. up a slope of 1 in 10 for a distance of 100 ft., the coefficient of friction being $\frac{1}{10}$, and the pulling force being exerted parallel to the plane.

Find also the work required to draw it down again through the same space.

7. If a body be let fall from a height of 64 ft. at the same instant that another is sent vertically upwards from the foot of the height with a velocity of 64 ft. per second, what time elapses till they meet?

If the first body starts 1 sec. later than the other, what time will elapse?

8. Find the time in which a body will fall down a given smooth inclined plane.

In a vertical circle, find the radius down which a particle will fall from the centre to the circumference in double the time of falling down the vertical radius.

9. Two particles describe the same circle with given different uniform velocities. Show that the line joining them turns with uniform angular velocity.

10. Two equal particles start simultaneously from the origin and move in the two rectangular axes, one with uniform velocity V , the other with uniform acceleration f , starting from rest. Find the equation of the path of their centre of gravity.

11. A body is projected from A in a given direction AB . If, when the projectile is at P , Q be the point of AB vertically above P , and M be the bisection of AQ , prove that the direction of its motion is MP .

12. A point P describes a circle with a given uniform velocity V . If Q be the projection of P on a given fixed diameter, show that the velocity of Q varies as PQ , and that its acceleration varies as QO , where O is the centre.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1889.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[The ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.]

1. ABC , DBC are two triangles on the same base and on the same side of it. If $AB = BD$, prove that AC cannot be also equal to CD .

2. Prove—

(1) That every parallelogram which has one angle a right angle, has all its angles right angles.

(2) That every quadrilateral which has all its sides equal, and one angle a right angle, has all its angles right angles.

*3. If a straight line be divided into any two parts, the squares on the whole line, and on one of the parts, are equal to twice the rectangle contained by the whole and that part, together with the square on the other part.

4. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

If C, D be the two points in which the straight line AB can be so divided, prove that the rectangle contained by AB, CD is equal to the rectangle contained by AC, CB .

5. If a straight line touch a circle, the straight line drawn from the centre to the point of contact shall be perpendicular to the line touching the circle.

Construct a triangle, having given the vertical angle, one of the sides containing it, and the perpendicular altitude.

6. The angles in the same segment of a circle are equal to one another.

AB is the base of a segment of a circle less than a semi-circle, and P any point on its circumference. AP is produced to Q , so that $BQ = BP$. Show that the locus of Q is a segment of an equal circle.

7. In a given circle inscribe a triangle equiangular to a given triangle.

8. Describe a circle about a given equilateral and equiangular pentagon. Prove that the diameter drawn through any angular point of the pentagon is perpendicular to the opposite side of the figure.

9. If the sides of two triangles, about each of their angles, be proportionals, the triangles shall be equiangular to one another, and shall have those angles equal which are opposite to the homologous sides.

10. If four straight lines be proportionals, the rectangle contained by the extremes is equal to the rectangle contained by the means; and if the rectangle contained by the means is equal to the rectangle contained by the extremes, the four straight lines are proportionals.

In the base AB of a triangle ABC a point D is taken so that the angle CDB is equal to the vertical angle ACB . Prove that CB touches the circle circumscribing the triangle ACD .

11. In any right-angled triangle, any rectilineal figure described on the side subtending the right angle is equal to the sum of the similar and similarly described figures on the sides containing the right angle.

II. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Divide $x(x^2 - yz) + y(y^2 - xz) + z(z^2 - xy)$ by $x + y + z$:
and find the value of $x^3 - x^2 + 3x + 5$, when $x = 1 + 2\sqrt{-1}$.

2. Find the H.C.D. of $30a^2x^4 - 5a^3x^3 + 5a^5x$
and $9ax^3 - a^3x + 2a^4$.

3. Show that if unity be added to the product of any four consecutive numbers the sum is a perfect square.

4. Show how to resolve any quadratic expression in X of the form $pX^2 + qX + r$ into factors.

Resolve $ac(x^2 + y^2) + xy(b^2 - c^2 - a^2) + ab(x^2 - y^2)$ into two factors, each containing first powers of x and y .

5. Prove that the square root of a rational quantity cannot be partly rational and partly a quadratic surd.

Find the square root of

$$3x - 1 + 2\sqrt{2x^2 + x - 6}.$$

6. Express $\frac{\sqrt{\frac{3}{2}} - \frac{2}{3}\sqrt{\frac{2}{3}}}{\sqrt{6} + \sqrt{\frac{2}{3}}}$ as a ratio of two whole numbers.

7. Solve the equations

$$(1) \quad \frac{8x-7}{4} + \frac{3x+7}{3x} + \frac{1}{2} = \frac{6x+1}{3}.$$

$$(2) \quad \sqrt{1 + \frac{bx}{a^2}} + \sqrt{1 - \frac{bx}{a^2}} = 1\frac{3}{5}.$$

$$(3) \quad \left. \begin{aligned} y^2 + xy &= 4 \\ x^2 + 2y^2 - xy &= 8 \end{aligned} \right\}.$$

8. Two companies of a regiment appeared in the field in strength as 9 to 11. At the end of the day the relative strength was 5 to 8. Of the men in the two companies 35 per cent. were missing, and 30 men of these belonged to the second company. Find the number in each company at first.

9. Prove that the Geometric mean between two quantities is a mean proportional to their Arithmetic and Harmonic means.

If the m^{th} term of an harmonic progression be n , and the n^{th} term be m , prove that the $(m+n)^{\text{th}}$ term will be $\frac{mn}{m+n}$.

10. If ${}^nC_r$ denote the number of combinations of n things taken r together, prove that

$${}^{n+1}C_r - {}^nC_r = {}^nC_{r-1}.$$

Eight prisoners have to be marched out each morning handcuffed in pairs. How many times can they be sent out without exactly repeating the arrangement of pairs, no account being taken of whether a prisoner is handcuffed by the left hand or the right or of the order in which the pairs march out?

11. Write down in their simplest forms the coefficients of x^{2r} in the expansions of $(1-x)^{-3}$ and $\frac{1+x}{(1-x^2)^3}$,

and determine the sum of the infinite series

$$1 + \frac{3}{2^3} + \frac{1 \cdot 3}{1 \cdot 2} \cdot \frac{3^2}{2^6} + \frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \cdot \frac{3^3}{2^9} + \text{etc.}$$

12. Given $\log_{10} 8 = \cdot 9030900$, $\log_{10} 28 = 1 \cdot 4471580$,
find $\log_{10} 350$ and $\log_{12 \cdot 5} 10$.

III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy.] Questions 7, 8, and 9 may be worked with the aid of the logarithmic tables supplied.]

1. Prove by means of geometrical figures that

$$\sin(180^\circ + A) = -\sin A,$$

and

$$\cos 330^\circ = \frac{\sqrt{3}}{2};$$

also find the value of

$$\cos 1410^\circ.$$

2. Prove that

$$\sin \theta = \frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}}.$$

Having given $\tan \theta = \frac{3}{4}$, find $\sin \theta$, $\cos \theta$, and $\operatorname{versin} \theta$.

3. Prove the formulæ,

$$(\alpha) \quad \cot A - \operatorname{cosec} 2A = \cot 2A;$$

$$(\beta) \quad 1 + \cos^2 2A = 2(\sin^4 A + \cos^4 A);$$

$$(\gamma) \quad \frac{\sin(A + 3B) + \sin(3A + B)}{\sin 2A + \sin 2B} = 2 \cos(A + B).$$

4. By means of a geometrical figure, or otherwise, prove that

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

If

$$\tan B = \frac{n \sin A \cos A}{1 - n \sin^2 A},$$

prove that

$$\tan(A - B) = (1 - n) \tan A,$$

and solve the equation

$$\tan x = \frac{3 \sin 2x \cos 2x}{1 - 3 \sin^2 2x}.$$

5. Find an expression for all the angles which have a given tangent.

If the value of $\tan 2\theta$ is given, determine the number of values of $\sin \theta$.

Prove that

$$\tan 67^\circ 30' = 1 + \sqrt{2}.$$

6. Prove that

$$\cos^{-1}x + \cos^{-1}y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}),$$

and solve the equation

$$\tan^{-1}\frac{x+1}{x-1} + \tan^{-1}\frac{x-1}{x} = \tan^{-1}(-7).$$

7. Prove that, in a triangle ABC ,

$$\tan \frac{1}{2}(B - C) = \frac{b-c}{b+c} \cot \frac{A}{2}.$$

Having given $b = 130$, $c = 63$, and $A = 42^\circ 15' 30''$, find the other angles, and the third side of the triangle.

8. An observer finds that from the doorstep of his house the angular elevation of the top of a church spire is 5α , and that from the roof above the doorstep it is 4α . The height of the roof above the doorstep being h , prove that the height of the top of the spire above the doorstep is equal to

$$h \operatorname{cosec} \alpha \cdot \cos 4\alpha \cdot \sin 5\alpha,$$

and that the horizontal distance of the top of the spire from the house is equal to

$$h \operatorname{cosec} \alpha \cdot \cos 4\alpha \cdot \cos 5\alpha.$$

If h is 39 feet and if α is equal to $7^\circ 17' 39''$, calculate the height and the distance.

9. Taking $\pi = 3.14159$, calculate approximately the area and the perimeter of the circle inscribed in a square, the side of which is 359.5678 feet.

10. A vessel in the shape of a circular cylinder, open at the top, the height of which is double its diameter, stands on a horizontal table, and is filled with water. A solid circular cone, of the same base and the same height, is pushed into the water until its vertex reaches the base of the cylinder, and is then taken out. Find the height at which the water afterwards stands in the cylinder.

If a solid sphere, of very nearly the same diameter as the cylinder, is then pushed in, and held so as to be wholly immersed, find the height at which the water will stand.

IV. STATICS AND DYNAMICS.

(Obligatory.)

1. Enunciate the "Triangle of Forces," and deduce an expression for the resultant of two forces P and Q acting at an angle θ .

If $P = 3$ lbs., $Q = 5$ lbs., and their resultant $= 7$ lbs., find the angle between P and Q .

2. A small heavy ring A which can slide upon a smooth vertical hoop is kept in a given position by a string AB , B being the highest point of the hoop. Show that the pressure between the ring and the hoop is equal to the weight of the ring.

3. A light rod AB , 20 inches long, rests upon two pegs whose distance apart is equal to half the length of the rod. How must it be placed so that the pressures on the pegs may be equal when weights $2W$, $3W$ are suspended from A , B respectively?

4. Define "the moment of a force," and explain what is meant by the algebraical sum of the moments of a system of forces in one plane about a point in that plane.

$ABCD$ is a square, the length of each side being 4 feet, and four forces act as follows: 2 lbs. from D to A , 3 lbs. from B to A , 4 lbs. from C to B , and 5 lbs. from D to B . Find (to the nearest foot-pound) the algebraical sum of the moments of the forces about C .

5. Describe the wheel and axle, and obtain the relation between P and W .

6. Show how to find the centre of gravity of a body composed of two parts, whose centres of gravity and weights are given.

A uniform one-foot rod is broken into two parts of 5 and 7 inches, which are then placed so as to form the letter **T**, the longer portion being vertical; find the position of its centre of gravity.

7. Define velocity and acceleration, and show how they are measured when uniform.

A body moving with uniform acceleration has, 3 seconds after starting, a velocity of 18 feet a minute; when will it be moving at the rate of 45 miles an hour?

8. A particle is projected vertically upwards with a velocity of 50 feet per second. Find (i.) when its velocity will be 25 feet per second, and (ii.) when it will be 25 feet above the point of projection.

9. ABC is a triangle, right-angled at C . Prove that, if it be placed with A uppermost, and AB vertical, the times of falling down AB , AC will be equal; and if placed with AC vertical the velocities acquired in falling down the same sides will be equal.

10. Explain clearly the meanings of the symbols employed in the equation $P = Mf$.

A force equal to the weight of 23 lbs. acts upon a mass of 41 lbs.; find the velocity acquired at the end of 3 seconds from rest.

11. A particle is constrained to describe a circle on a smooth horizontal table by means of a string 18 inches long, having one end attached to a fixed point of the table. If the tension of the string be equal to the weight of the particle, find the velocity.

12. A stone is let fall from the top of the mast of a ship which is sailing with uniform velocity (25 feet per second). Represent in a figure, and describe the positions of the mast and the stone at the end of the first, second, and third seconds, and thence draw (roughly) the actual path of the stone, neglecting the resistance of the air.

(You can use the whole length of your page for this diagram.)

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[Full marks may be obtained for about five-sixths of this paper. Great importance is attached to accuracy.]

1. Define a conic section as the locus of a point which moves subject to certain conditions.

Show that if a line be drawn from the point where the axis meets the directrix through the extremity of the latus rectum, its inclination to the axis will be greater than, equal to, or less than 45° according as the curve is a hyperbola, parabola, or ellipse.

2. Prove that in a parabola tangents at the extremities of a focal chord intersect at right angles in the directrix.

AB is the chord of a circle ABC , show that the centres of all the circles touching AB and ABC will trace out two parabolas, and draw a figure showing both parabolas.

3. If the tangent at P meet the major axis of an ellipse in T , and N be the foot of the ordinate PN , prove that the rectangle contained by CN and CT is equal to the square on AC .

From any point P on an ellipse, the ordinate PN and normal PK are drawn to meet the major axis in N and K , show that the ratio of CN to KN is the same for all positions of P .

4. Define conjugate diameters of an ellipse. If CP and CD be conjugate, show that

$$CP^2 + CD^2 = AC^2 + BC^2.$$

5. If from any point P on an hyperbola the ordinate PN is drawn, prove that

$$PN^2 : AN \cdot A'N :: BC^2 : AC^2.$$

If PN be the ordinate of any point P on a rectangular hyperbola, and from N a tangent NP' is drawn to the point P' on the auxiliary circle, prove that $NP = NP'$.

6. A right cone is cut by a plane, which is parallel to a line in its surface and perpendicular to the plane containing that line and the axis: show that the section is a parabola.

Prove that the latus rectum of this section varies as the distance between the vertices of the cone and the parabola.

7. Obtain the equation to a straight line in terms of the length of the perpendicular upon it from the origin and the angle which that perpendicular makes with one of the rectangular axes. Find the area of OLL' , where L, L' are the feet of the perpendiculars let fall from the origin O , upon the lines

$$x \cos \alpha_1 + y \sin \alpha_1 = p_1, \quad x \cos \alpha_2 + y \sin \alpha_2 = p_2.$$

8. Trace the circle

$$(x^2 + y^2) + a^2x + b^2y + c^3 = 0,$$

and show that if α, β be the coordinates of the centre, then

$$\frac{a}{\alpha^2} = \frac{\beta}{b^2}.$$

9. Define the normal of any curve. Find the equation to the normal at any point of a parabola, reducing it to a form suitable for demonstration of the theorem that from any point there cannot be drawn more than three normals to a parabola.

10. Show that the equations $x = a \cos \phi$ and $y = b \sin \phi$ represent an ellipse, where ϕ is a variable and a, b are constants.

P and p are corresponding points on an ellipse and auxiliary circle, with centre C ; two parabolas with their vertices at C , and axes coincident with the major axis of the ellipse, are drawn passing through P and p ; show that if l, l' are the latera recta of the parabolas then $\frac{l}{l'} = 1 - e^2$, where e is the eccentricity of the ellipse.

II. Find the polar equation of a conic, the focus being the pole; and show that in any conic the semi-latus rectum is a harmonic mean between the segments of any focal chord.

VI. PURE MATHEMATICS (2).

[Full marks may be obtained for about three-fourths of this paper. Great importance is attached to accuracy.]

1. Find the value of z from the equations

$$x^2 + z = a, \quad 2xy + z = b, \quad y^2 + z = c.$$

2. Find a number divisible by 17 which, when divided by 7, leaves 4 as remainder.

3. How many terms are there in the expansion of

$$(a + b + c + d)^4?$$

Give the coefficient of the term containing $abcd$.

4. In how many ways may 16 people be divided into 4 groups of 4 persons in each?

5. Find values (different from a, b) for A, B , which will render the equation

$$m(x + A)^2 + n(x + B)^2 \equiv m(x + a)^2 + n(x + b)^2$$

an identity for all values of x .

6. If $mx^2 + ny^2 = c$, where m, n are positive, show algebraically that $x + y$ is greatest when $mx = ny$; and give its greatest value in terms of m, n, c .

7. If $\sin x + \cos x = \sin a + \cos a$,
state all the possible values of x .

8. Resolve $x^4 - 2x^2 \cos \theta + 1$
into two real quadratic factors.

9. A quadrilateral $ABCD$ is circumscribed about a circle; prove that
 $AB \sin \frac{1}{2}A \sin \frac{1}{2}B = CD \sin \frac{1}{2}C \sin \frac{1}{2}D$.

10. Find the sum of the infinite series
 $\sin \theta - \frac{1}{2} \sin 2\theta + \frac{1}{3} \sin 3\theta - \text{etc.}$
and deduce from it the sum when all the signs are $+$.

11. If $x^3 + y^3 + ax^2 + bxy + cy^2$
can be decomposed into two real factors of the 1st and 2nd degrees, prove
that either $a + c = b$, or $a = c = -b$.

VII. MECHANICS.

[Full marks may be obtained for about three-fourths of the paper. Mathematical instruments must be used for questions requiring a graphical method of solution.]

1. Enunciate the proposition known as the Polygon of Forces and employ it to solve graphically the following problem.

Four strings are knotted together at O ; of these, two, viz. OA, OB , have their extremities A, B fixed, a third hangs vertically and supports a weight of 20 lbs., and the remaining one passes over a pulley at C , (ACB being a horizontal line), and supports a weight of 5 lbs. :

$$AC = 2\frac{1}{2} \text{ in.}, \quad CB = 2\frac{1}{2} \text{ in.}, \quad OA = 4\frac{1}{2} \text{ in.}, \quad OB = 2\frac{1}{2} \text{ in.}$$

Find the tensions of the strings OA, OB .

2. Solve graphically the following problem.

An iron ring, outside diameter 3'96 in., thickness '32 in. and weighing 3 lbs. rests in a vertical plane in contact with two perfectly smooth equal pegs, diameter '34 in., whose centres are in the same vertical line and 3 in. apart. The upper peg is inside and the lower peg is outside the ring.

Find the pressures of the ring upon the pegs.

3. State the conditions of equilibrium in the case of a system of forces acting in one plane on a rigid body.

A cylindrical vessel, whose height is 5 inches and diameter 3 inches, stands upon a horizontal plane and a smooth uniform rod 9 inches long is placed within it resting against the edge. Find the pressures between the rod and the vessel, the weight of the former being 6 ozs.

4. Express the coordinates of the centre of gravity of a system of heavy particles lying in a plane in terms of the weights and coordinates of the particles.

If G be the centre of gravity of a system of equally heavy particles $A, B \dots$ prove that the forces represented by GA, GB , etc. are in equilibrium.

5. Two equal moveable pulleys A, B , each of weight w , have their centres connected by a string which passes round a fixed pulley C , of the same diameter as A or B , and a second string having one end fixed, passes under A and over B , all the strings being parallel. What force P must be applied to the free end of this string to balance W suspended from the pulley A ?

Show that for a vertical displacement of the system $P \times P$'s virtual velocity $+ W \times W$'s virtual velocity $= 0$.

6. Explain the terms "coefficient of limiting friction," "work."

A weight of 2 cwt. rests on a rough plane ($\mu = \frac{1}{4}$) inclined to the horizon at an angle of 30° , and a string attached to it passes over a smooth pulley at the summit and, hanging freely, supports a weight of 1 cwt. A vertical force is applied to this latter weight just sufficient to begin to raise it. Find in foot-pounds the work done in thus raising it through 50 feet.

7. A particle starts from a point O with uniform velocity 4 feet per second, and after 2 seconds another particle leaves O in the same direction with a velocity of 5 feet per second, and subject to an acceleration whose measure is 3. Find when and where it will overtake the first particle.

8. Two small rings (connected by a string which remains tight throughout the motion) move along two wires which intersect at right angles. Show that the point midway between them describes a circle and that at any moment the velocities of the rings are inversely as their distances from the point of intersection of the wires.

9. Define the unit of acceleration. Two weights are connected by a string which passes round a smooth pulley. Find their ratio in order that they may move with the unit acceleration, one second and one foot being the units of time and length.

10. Two marbles of equal diameter but of masses $10m$, $11m$, are projected from the same point, with equal velocities but in opposite directions, along a circular groove; where will the second impact take place, e being $= \frac{3}{4}$?

11. A pendulum 3 feet long is observed to make 700 oscillations in 671 seconds; find approximately the value of g .

12. A particle is projected with velocity u in a direction making an angle α with the horizon; investigate the equation of the path described.

Two particles projected with the same velocity from O , pass through the same point P , show that if α , β be the angles of projection

$$\alpha + \beta = \frac{\pi}{2} + i,$$

where i = angle OP makes with the horizon.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1890.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*The ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Draw a perpendicular to a given line from a given point outside it.

2. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of any triangle are together equal to two right angles.

Draw a straight line DE parallel to the base BC of a triangle to cut the side AB in D and AC in E , so that DE may be equal to BD and CE together.

3. D is a point in the side AB of a triangle. Find a point E in the side BC such that the triangles EAD , CAE may be equal.

4. In any right-angled triangle the square which is described on the side subtending the right angle is equal to the sum of the squares described on the sides which contain the right angle.

Make a square which is three times the square on a given straight line.

WOOLWICH ENTRANCE EXAMINATION.

5. Divide a straight line so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

6. Give a geometrical proof of the algebraic formula

$$(a+b)^2 + (a-b)^2 = 2(a^2 + b^2).$$

7. If a point be taken within a circle from which there fall more than two equal straight lines to the circumference, that point is the centre of the circle.

8. If a straight line touch a circle and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle.

AB is trisected in the points *C*, *D*, and on *CD* an equilateral triangle *CPD* is described. Show that a circle passing through the three points *B*, *C*, *P*, will touch the straight line *AP*.

9. Describe an isosceles triangle having each of the angles at the base double of the third angle.

If *ABD* be the triangle required and *C* the point in the side *AB* used in Euclid's construction, and if *DC* be produced to meet the larger circle in *F*, and *FB* be joined, prove that *FB* is a side of a regular pentagon inscribed in the larger circle.

10. If two triangles have one angle of the one equal to one angle of the other, and the sides about the equal angles proportional, the triangles shall be equiangular, and shall have those angles equal which are opposite to the homologous sides.

11. Define duplicate ratio, and prove that similar triangles are to one another in the duplicate ratio of their homologous sides.

12. Construct two equilateral triangles which shall have a given ratio to each other and which shall be together equal to a given equilateral triangle.

II. ALGEBRA.

(Up to and including the Binomial Theorem; the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Divide $2x^5 - 10x^4y + 9x^3y^2 + 13x^2y^3 - 18xy^4 + 3y^5$ by $2x^2 - 3y^2$.

2. Find the highest common factor of

$$x^3 - 6a^2x - 9a^3, \quad \text{and} \quad 2x^3 - 10ax^2 + 9a^2x + 9a^3.$$

3. Find the value of $(\sqrt{5} + \sqrt{3})^4 + (\sqrt{5} - \sqrt{3})^4$, and prove that 4 is one of the values of $(161 + 72\sqrt{5})^{\frac{1}{4}} + (161 - 72\sqrt{5})^{\frac{1}{4}}$.

4. Solve the equations

$$(i.) \quad \frac{x+2}{x-4} + \frac{x+4}{x-2} = 2;$$

$$(ii.) \quad (a-b)x + (a+b)y = a^2 - b^2.$$

$$(a+b)x - (a-b)y = 2ab.$$

5. Prove that a quadratic equation cannot have more than two roots.

Form the equation of which the roots are

$$\frac{1}{3}(5 + \sqrt{-7}) \quad \text{and} \quad \frac{1}{3}(5 - \sqrt{-7}).$$

6. Solve the equations

$$(i.) \quad 15x^2 - 76x + 96 = 0;$$

$$(ii.) \quad 27x^3 + y^3 = 341, \quad 3x + y = 11.$$

7. The number of men required to form a hollow square is three times the number required to fill up the interior empty space. Prove that the length of each face of the hollow square is four times its depth.

8. Prove that $(x+y+z)^5 - x^5 - y^5 - z^5$ is divisible by

$$(y+z)(z+x)(x+y),$$

and find the other factor.

WOOLWICH ENTRANCE EXAMINATION.

9. Insert five harmonic means between 2 and $\frac{2}{13}$.

If a, b, c are positive quantities in geometrical progression, prove that

$$a^2 + c^2 > 2b^2.$$

10. Find an expression for the number of permutations of n things taken r together.

In how many of these will two given things occur in juxtaposition?

11. Write down the co-efficients of x^r in the expansions of $(1-x)^{-n}$, and $(1-x)^{-\frac{1}{2}}$, and prove that the co-efficient of x^r in the expansion of $\left(\frac{1-x}{1-2x}\right)^2$ is $(r+3)2^{r-2}$.

12. Define the logarithm of a number to a given base, and prove that

$$\log_a \frac{m}{n} = \log_a m - \log_a n.$$

Determine, by aid of the tables, the logarithm, to the base 10, of the fraction

$$\frac{5742.96}{43.793}$$

13. Employ the tables to solve the equation,

$$(255)^x = (374)^{x-1}.$$

III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy. Questions 7, 8, and 9 may be worked with the aid of the logarithmic tables supplied.]

1. Explain the various ways of measuring angles. Calculate, in degrees, minutes, and seconds, the angle whose circular measure is unity.

2. Prove that

$$(1) \tan 60^\circ = \sqrt{3}.$$

$$(2) \cos 2A = 1 - 2 \sin^2 A.$$

3. Prove that

$$2 \cos A = \pm \sqrt{1 + \sin 2A} \pm \sqrt{1 - \sin 2A}.$$

Determine the signs in the case when A is a negative angle between -45° and -90° .

4. Express $\sin A$ in terms of $\sin \frac{A}{3}$.

Why is it natural to expect that the result will be of the third degree in $\sin \frac{A}{3}$?

5. In any triangle prove the following relations—

$$(1) \cot A + \cot B = \frac{c}{b} \operatorname{cosec} A.$$

$$(2) \frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B} = 2.$$

$$(3) \cos(B - C) \sin 3A + \cos(C - A) \sin 3B + \cos(A - B) \sin 3C = 0.$$

6. Explain the inverse notation of trigonometrical functions.

If $\operatorname{cosec}^{-1} x = \operatorname{cosec}^{-1} a + \operatorname{cosec}^{-1} b$, find the value of x in terms of a and b .

WOOLWICH ENTRANCE EXAMINATION.

7. Show how to solve a triangle when two angles and a side opposite to the third angle are given.

From the top of a hill the angles of depression of two successive milestones on level ground, and in the same vertical plane as the observer, are found to be 5° and 10° respectively. Find the height of the hill and the horizontal distance to the nearest milestone.

8. Write down formulæ adapted to the use of logarithms for solving a triangle when two sides and the included angle are given.

Solve the triangle in which

$$a = 242.5, \quad b = 164.3, \quad C = 54^\circ 36'.$$

9. State formulæ for the areas of the curved surfaces of a sphere, of a right circular cylinder, and of a right circular cone.

How many square yards of canvas are required to make a conical tent 9 feet high, such that a man of 6 feet could stand anywhere inside, within a radius of 2 feet from the centre without stooping?

10. A pint tankard is in the form of a frustum of a circular cone; its height is $4\frac{1}{2}$ inches and the diameter of its base $3\frac{1}{2}$ inches, both measurements taken inside. Find the diameter of the top, being given that a gallon of water weighs 10 lbs. and a cubic foot of water weighs 1000 ozs. Employ logarithm tables to find the dimensions of the similar quart tankard to four places of decimals.

IV. STATICS AND DYNAMICS.

(Obligatory.)

1. Show that forces may be fully represented by straight lines.

The length of the side of a square $ABCD$ is one foot, and in the sides CB , CD points E , F are respectively taken such that $CE = 7$ inches and $CF = 9$ inches. Draw a figure showing a particle acted on by forces represented by AE , EB , BF , FD , DA , and state the magnitudes of the forces if 5 inches represent a force of 2 lbs.

2. A particle of weight W is supported within a smooth hemispherical bowl by a string of given length having one end attached to a point in the rim. State clearly the forces which keep the particle in equilibrium and find their magnitudes if the length of the string = the radius of the bowl, and the rim is in a horizontal plane.

3. Show that if two forces act on a body the algebraical sum of their moments about any point in their plane is equal to the moment of their resultant about that point.

Find the moment about D of the resultant of the two forces represented by BA , BF in Question 1, stating in what units the moment is measured.

4. State the rule for finding the magnitude and line of action of two like parallel forces.

AB is a rod one foot in length; when a weight of $\frac{1}{2}$ lb. is suspended from A , the rod balances about a point 3 inches from A , and when the same weight is suspended from B , it balances about a point 5 inches from B . Find the weight of the rod and the position of its centre of gravity.

5. Find the centre of gravity of a triangular lamina ABC , and show that it coincides with that of three equal particles placed at the angular points.

If the particle at A be moved to A' , the foot of the perpendicular from A on BC , prove that the centre of gravity will move to G' , the foot of the perpendicular from G on BC .

6. Investigate the relation between the power and the weight in the case of the smooth inclined plane.

If the inclination of the plane to the horizon be 30° , and the power be applied by means of a string passing over a fixed pulley, find the ratio of the power to the weight in order that in the position of equilibrium the portion of the string between the weight and the pulley may make equal angles with the inclined plane and the vertical.

7. Enunciate and prove the Parallelogram of Velocities.

A man rows across a river $\frac{1}{4}$ of a mile broad in 5 minutes always keeping his boat at right angles to the current. On reaching the opposite side he finds he is $\frac{1}{2}$ a mile from the starting point. Find the velocity of the current.

8. Define uniform acceleration and from your definition obtain the formula $v = u + ft$.

The motion of a particle is uniformly accelerated, and at the beginning and end of a certain interval of 5 seconds the velocities are known to be 3 and 7 feet per second respectively ; find the space described in the given interval.

9. Write out Newton's Three Laws of Motion, and give an illustration of each. Deduce from them the relation between the acceleration of a given mass and the force producing it.

10. Find the force which acting on a mass of 6 lbs. will cause it to describe 9 feet in 2 seconds from rest. What will be the measure of this force if the unit of force be the weight of one ounce ?

11. A particle moves with uniform velocity inside a smooth horizontal circular tube one foot in diameter at the rate of seven revolutions per minute. Compare the horizontal pressure on the tube with the weight of the particle.

12. A body is projected with a velocity u in a direction making an angle α with the horizon ; find the greatest height above the horizontal plane through the point of projection attained by the body, and also the range on this plane.

Find the least velocity of projection which will give a range of 1,200 yards.

FURTHER EXAMINATION.

V. PURE MATHEMATICS (1).

[Full marks may be obtained for about three-fourths of this paper. Great importance is attached to accuracy.]

1. Define a conic section, and describe the respective positions of the focus and directrix, (1) when the conic is a circle, (2) when it consists of two straight lines, either parallel to, or intersecting one another.

2. If the tangent at any point P of a conic meets the directrix in F , and the latus rectum produced in D prove that FSP is a right angle, and that

$$SD : SF :: SA : AX.$$

If a conic of given eccentricity is drawn touching the straight line FD joining two fixed points F and D , and if the directrix always passes through F , and the corresponding latus rectum always passes through D , find the locus of the focus.

3. If PN is the ordinate of the point P of a parabola, of which A is the vertex, and S the focus, prove that

$$PN^2 = 4AS \cdot AN.$$

If the straight line AP and the diameter through P meet the double ordinate QMQ' in R and R' , prove that

$$RM \cdot R'M = QM^2.$$

4. Prove that the tangents drawn from an external point to an ellipse are equally inclined to the focal distances of that point.

The tangent at the point P of an ellipse meets the directrices in E and F ; prove that the other tangents from E and F intersect on the normal at P .

5. Prove that the asymptotes of an hyperbola intersect the directrices in the same points as the auxiliary circle, and that the lines joining the corresponding foci with the points of intersection are tangents to the circle.

6. Prove that any section of a right circular cylinder by a plane not parallel or perpendicular to the axis of the cylinder is an ellipse, and that the foci are the points of contact of spheres inscribed in the cylinder and touching the plane. What is the locus of the foci of all sections of the same eccentricity?

7. Trace the points, straight lines, or curves, respectively represented by the equations,

$$(i.) (x^2 - 1)^2 + (y^2 - 4)^2 = 0;$$

$$(ii.) (x^2 - 1)(y^2 - 4) = 0;$$

$$(iii.) y^4 = 16a^2x^2;$$

$$(iv.) xy = a(x + y).$$

8. Find the polar equation of a straight line in the form

$$\rho = r \cos(\theta - \alpha).$$

If a triangle ABC remain similar to a given triangle, and if the point A be fixed, and the point B move along a straight line, find the locus of the point C .

9. Find the equation of the circle, having its centre at the point (α, β) and passing through the origin, and prove that the equation of the tangent at the origin is

$$\alpha x + \beta y = 0.$$

10. Find the equation of the tangent at a point of the parabola, $y^2 = 4ax$, in the form

$$y = mx + \frac{a}{m}.$$

Determine the locus of the point of intersection of two tangents to the parabola, which are inclined to each other at the angle of 45° .

11. Obtain the equation of the normal at the point $(a \cos \phi, b \sin \phi)$ of the ellipse $a^2y^2 + b^2x^2 = a^2b^2$, in the form

$$\frac{ax}{\cos \phi} - \frac{by}{\sin \phi} = a^2 - b^2.$$

If ψ be the eccentric angle of the other point where this normal intersects the ellipse, prove that

$$\tan \frac{1}{2}(\psi + \phi) = -\frac{b^2}{a^2} \cot \phi.$$

12. Show how to find the coordinates (h, k) of the centre of a curve of the second order $f(x, y) = 0$, and prove that the asymptotes are given by the equation $f(x, y) = f(h, k)$.

Show that the asymptotes of the curve

$$y^2 - xy - 2x^2 - 5y + x - 6 = 0$$

are

$$y + x = 2 \text{ and } y - 2x = 3.$$

VI. PURE MATHEMATICS (2).

[Full marks may be obtained for about three-fourths of this paper. Great importance is attached to accuracy.]

1. If

$$\frac{a-p}{b-p} + \frac{a-q}{b-q} = 0,$$

show that

$$\frac{1}{(a-p)(b-p)} + \frac{1}{(a-q)(b-q)} + \frac{4}{(a-b)^2} = 0.$$

2. Find the sum of all the numbers in the first 1,000

(1) which are not divisible by 3 ;

(2) which are not divisible by either 2 or 5.

3. In how many ways can a man distribute 5 votes amongst 5 candidates, it being allowable to give more than one vote to any candidate ?

4. Give short methods for discovering if a number is divisible by 3, 6, and 9 respectively.

Show that a number is divisible by 2^n if the number represented by the last n digits is so divisible.

5. Show how to solve in integers an indeterminate equation of the first degree in two variables.

Divide 100 into two integer parts, divisible by 7 and 11 respectively.

6. The probability of A winning a game with B is p and with C is p' .

Find the probability that A will defeat B and C one after the other.

Show that the chance of B winning a game with C is

$$\frac{p'(1-p)}{p(1-p') + p'(1-p)}.$$

7. If lines drawn from the angular points A, B, C of a triangle so as to bisect the opposite sides, make angles α, β, γ with the sides AB, BC, CA respectively; show that

$$\cot \alpha + \cot \beta + \cot \gamma = 3(\cot A + \cot B + \cot C).$$

8. Solve completely the equation

$$\cos x + \cos 7x = \cos 4x,$$

and write down all the angles between 180° and 360° which satisfy it.

9. Employ De Moivre's theorem in Trigonometry to obtain the cube roots of unity. If ω be either of the imaginary roots, show that $1 + \omega + \omega^2 = 0$.

Exhibit $x^2 - xy + y^2$ as the product of its imaginary linear factors.

10. An observer in the first place stations himself at a horizontal distance a feet from a column standing upon a mound. He finds that the column subtends an angle $\tan^{-1} \frac{1}{2}$ at his eye, which may be supposed on the horizontal plane through the base of the mound. On moving $\frac{3}{4}a$ feet nearer the column he finds the angle subtended unchanged. Find the height of the mound and of the column.

11. Find the sums to n terms of the series

(1) $\sin \theta + \sin(\theta + \alpha) + \sin(\theta + 2\alpha) + \dots$

(2) $\sin^2 \theta + \sin^2(\theta + \alpha) + \sin^2(\theta + 2\alpha) + \dots$

VII. MECHANICS.

[Full marks may be obtained for about three-fourths of the paper. Mathematical instruments must be used for questions requiring a graphical method of solution.]

1. Two forces are represented in magnitude and direction by two straight lines AB , CD ; show that their resultant is represented by twice the line joining the middle points of AC and BD .

2. Solve graphically the following problem.

A uniform triangular lamina ABC of 3 lbs. weight can turn in a vertical plane about a hinge at B ; it is supported with the side AB horizontal by a prop at the middle point of BC . Find the pressure on the prop and the strain on the hinge.

$$AB = 3 \text{ in.}, \quad BC = 2\frac{1}{2} \text{ in.}, \quad AC = 2 \text{ in.}$$

3. Explain what is meant by the "coefficient of limiting friction," and show how to determine from it the direction of the mutual resistance between two bodies, when slipping is on the point of taking place between them.

Solve graphically the following problem.

A uniform rectangular block $ABCD$ of 40 lbs. weight rests upon a rough horizontal plane CD ($\mu = \frac{1}{2}$), and a horizontal force of 10 lbs. acts at D ; find the magnitude of the least force P which, applied at the middle point of BC parallel to the diagonal DB , will move the block.

$$AB = 1\frac{1}{2} \text{ in.}, \quad BC = .8 \text{ inches.}$$

4. Two forces P , Q make angles α , β with a third force R upon opposite sides of it; find the magnitude and direction of their resultant.

If $\alpha = \beta$, and $R = 2\sqrt{PQ}$ show that the resultant $= P + 2\sqrt{PQ} \cdot \cos \alpha + Q$.

5. Show that if two couples are in equilibrium their moments must be equal but of opposite sign.

A thin ring of radius R and weight W is placed round a vertical cylinder of radius r , and is prevented from falling by a nail projecting horizontally from the cylinder. Find the pressure between the cylinder and the ring.

6. A uniform ladder rests at an angle of 45° with the horizon with its upper extremity against a rough vertical wall and its lower extremity on the ground. If μ , μ' be the coefficients of limiting friction between the ladder and the ground and wall respectively, show that the least horizontal force which will move the lower extremity towards the wall

$$= \frac{1}{2} W \cdot \frac{1 + 2\mu - \mu\mu'}{1 - \mu'}.$$

WOOLWICH ENTRANCE EXAMINATION.

7. Assuming the relation $s = ut + \frac{1}{2}ft^2$, find an expression for the velocity at any moment in terms of u , f , and s .

The greatest height attained by a particle projected vertically upwards is 225 feet; find how soon after projection the projectile will be at a height of 176 feet.

8. In the case of a single moveable pulley the free end of the string passes round a fixed pulley and supports a weight P greater than $\frac{1}{2}W$, where W = the weight suspended from the moveable pulley. Find the tension of the string during the ensuing motion, the three parts into which it is divided by the pulleys being parallel.

9. Two equal particles start simultaneously from the origin and describe the straight lines $x + y = 0$, $x - 7y = 0$ with uniform velocities $9u$, $10u$ respectively, the motion of each particle being to the right of the axis of y . Show that the centre of gravity of the particles moves with uniform velocity $8\frac{1}{2}u$, and find the equation of its path.

10. An imperfectly elastic ball impinges with given velocity and in a given direction upon a smooth fixed plane; find the motion of the ball after impact.

Given the initial position of the ball, show that whatever be the velocity and direction of projection before impact the line of motion of the centre of the ball after impact will pass through a fixed point.

11. Find the velocity acquired by a heavy particle in sliding down a smooth curve.

A small heavy ring can slide upon a cord 34 feet long which has its ends attached to two fixed points A , B in the same horizontal line and 30 feet apart. The ring starts—the string being tight—from a point 5 feet from A ; show that, when it has described a length of the cord = 3 feet, its velocity will be 10.12 feet per second nearly.

12. Show that the path of a projectile in vacuo is a parabola and give a construction for finding the position of its focus.

A particle slides down a rough inclined plane AB ; show that, if F be the focus of the parabolic path described after leaving the plane, the angle $AFB = \frac{\pi}{2}$ + the angle of limiting friction.

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I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.]

1. The angles at the base of an isosceles triangle are equal to one another; and if the equal sides be produced the angles on the other side of the base shall be equal to one another.

2. If a straight line falling on two other straight lines, make the alternate angles equal to one another, the two straight lines shall be parallel to one another.

3. Define a *rhombus*; and show that a rhombus is a parallelogram, and that its diagonals are at right angles.

4. If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts together with twice the rectangle contained by the parts.

Show that the area of any rectangle $AHGC$ is half the area of the rectangle contained by the diagonals of the squares described on two adjacent sides of $AHGC$.

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5. In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

If squares $ABDE$, $ACFG$ be described outwards on the sides AB , AC of a triangle ABC ; show that the sum of the squares on EG and BC is double of the sum of the squares on AB and AC .

6. A segment of a circle being given, describe the circle of which it is a segment.

7. If two straight lines cut one another (*not at right angles*) within a circle, the rectangle contained by the segments of one of them shall be equal to the rectangle contained by the segments of the other.

8. If from the extremities of the diameter AB of a semicircle $AEDB$ chords ACD , BCE be drawn to meet the semicircle in D and E respectively; show that the square on the diameter is equal to the sum of the rectangles $AC \cdot AD$ and $BC \cdot BE$.

9. Inscribe a square in a given circle.

The circumference of each of two circles passes through the centre of the other; describe a square in the figure which bounds the space common to the two circles.

10. Inscribe an equilateral and equiangular pentagon in a given circle.

11. If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides, or those sides produced, proportionally.

Produce a straight line AB to C , so that the rectangle contained by AC and CB may be equal to the square on AB .

12. Similar triangles are to one another in the duplicate ratio of their homologous sides.

If the sides BC , CD , DA , AB of a quadrilateral $ABCD$ be trisected respectively in A_1 and A_2 , B_1 and B_2 , C_1 and C_2 , D_1 and D_2 ; show that the figure $A_1A_2B_1B_2C_1C_2D_1D_2$ is seven-ninths of the whole quadrilateral.

II. ALGEBRA.

(Up to and including the Binomial Theorem; the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Multiply $x^3 - 2x^2 + x - 1$ by $x^3 + 2x^2 - x - 1$, and divide $(x^2 - 5x + 6)(x^2 - 5x - 14)$ by $(x + 2)(x^2 - 10x + 21)$.

2. Simplify

$$\frac{(1-a^2)(1-b^2)(1-c^2) + (a-bc)(b-ac)(c-ab)}{1-abc}$$

and write

$$2(x-y)(x-z) + 2(y-z)(y-x) + 2(z-y)(z-x)$$

as the sum of three squares.

3. Find the numerical values of the expressions $x^3 + y^3 + 3xy(x+y)$, and $x^3 - y^3 - 3xy(x-y)$ when $x = 5.5$, $y = 4.5$.

4. Determine the highest factor which will divide each of the expressions

$$x^4 - 10x^2 + 9, \quad x^4 + 7x^3 + 11x^2 - 7x - 12, \quad \text{and} \quad x^4 + 2x^3 - 16x^2 - 2x + 15.$$

5. Solve the equations

$$(1) \quad \frac{x-1}{x-2} - \frac{x-2}{x-3} = \frac{x-4}{x-5} - \frac{x-5}{x-6};$$

$$(2) \quad (b-a)x - (c-b)y = c-a, \\ (a-c)x - (b-a)y = b-c.$$

6. Determine under what conditions the roots of the quadratic

$$px^2 + qx + r = 0$$

may be real and different, and find between what limits a must lie in order that the roots of

$$x^2 + a^2 = 6a - 8x$$

may be real.

7. Solve the equations

$$(1) \quad x^2 + (1 - a^2)x = a^2;$$

$$(2) \quad \begin{cases} x + y = 3 \\ \frac{x^3 + y^3}{x - y} = 9. \end{cases}$$

8. Two men, A and B , travel along a road 180 miles long in opposite directions, starting simultaneously from the ends of the road. A travels 6 miles a day more than B , and the number of miles travelled each day by B is equal to double the number of days before they meet. Find the number of miles which each travels in a day.

9. Obtain the formula for the sum of an infinite number of terms of a geometrical progression, the ratio being less than unity, and apply it to the determination of the numerical value of the recurring decimal $\cdot 2342$.

In any geometrical progression of an odd number of terms, prove that the sum of the extremes is numerically greater than twice the middle term.

10. Prove that the number of combinations of $p + q$ different things taken p together, is the same as the number taken q together.

A cricket club consists of 13 members, of whom only 4 can bowl. In how many ways can an eleven be chosen so as to include *at least* 2 bowlers?

11. Write down the expansion of $\frac{1}{(1-x)^4}$ in ascending powers of x as far as 6 terms, and determine the value of the infinite series

$$1 + \frac{3}{2^3} + \frac{1 \cdot 3}{1 \cdot 2} \frac{3^2}{2^6} + \frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \frac{3^3}{2^9} + \text{etc.}$$

Prove that if $\sqrt{\frac{a+x}{a-x}}$ be expanded in ascending powers of $\frac{x}{a}$, the coefficients of the $(2r-1)^{\text{th}}$ and $2r^{\text{th}}$ terms are equal.

12. Given a series of logarithms of numbers calculated to base e , determine the modulus by which each must be multiplied to transform them all to base 10.

Employ logarithmic tables, and determine (1) the value of $\log_2 10$; (2) the number of years in which a given sum will double itself if put out at compound interest at the rate of 4 per cent. per annum.

III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Prove that $\sin 30^\circ = \frac{1}{2}$, and find the values of $\sin 210^\circ$ and $\sin 750^\circ$.

2. Taking $A + B$ less than 90° , prove by a geometrical figure that

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

Prove that $4 \sin^2 75^\circ = 2 + \sqrt{3}.$

3. Find $\tan A$ in terms of $\tan 2A$, and show, *à priori*, that two values will be obtained.

Prove that $\tan 7^\circ 30' = \sqrt{6} + \sqrt{2} - \sqrt{3} - 2.$

4. Solve the equation

$$\sin 11\theta + \sin 5\theta = \sin 8\theta.$$

5. Prove that, for a triangle ABC ,

$$(1) \quad a = b \cos C + c \cos B.$$

$$(2) \quad \frac{b^2 - c^2}{\tan A} + \frac{c^2 - a^2}{\tan B} + \frac{a^2 - b^2}{\tan C} = 0.$$

6. Prove that the distances of the centre of the inscribed circle of a triangle ABC from the centres of the escribed circles are respectively

$$a \sec \frac{A}{2}, \quad b \sec \frac{B}{2}, \quad c \sec \frac{C}{2}.$$

7. Solve the equation

$$\tan^{-1}(x + 1) + \tan^{-1}(x - 1) = \tan^{-1} \frac{8}{3};$$

and prove that

$$\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3) = 15.$$

8. Prove that, in a triangle ABC ,

$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

If $a = 21$, $b = 23$, and $c = 32$, calculate the angle A .

9. Looking out of a window, with his eye at the height of 15 feet above the roadway, an observer finds that the angle of elevation of the top of a telegraph post is $17^{\circ} 18' 35''$, and that the angle of depression of the foot of the post is $8^{\circ} 32' 15''$; calculate the height of the telegraph post, and its distance from the observer.

10. Prove that the area of a sector of a circle is equal to half the product of the length of the arc and the radius.

If the area is 2240.567 square feet, and the radius 33.495 feet, calculate the length of the arc.

11. An iron boiler is constructed in the form of a cylinder with hemispherical ends. If the radius of the boiler is one foot, and its extreme length (inside) four feet, calculate, as a fraction of a ton, the weight of water which it will hold, assuming that 1000 oz. is the weight of a cubic foot of water.

IV. STATICS AND DYNAMICS.

(Obligatory.)

1. Enunciate the "Parallelogram of Forces," and assuming it true for the direction of the resultant prove it for the magnitude.

Two forces are represented in magnitude and direction by OA and OB ; show that the resultant is represented by $3OC$, where C is one of the points of the trisection of AB .

2. A, B, C are three points in a vertical plane, A and C lying on opposite sides of the vertical line through the highest point B . A string $ADBCDE$ having one end fixed at A passes in succession through a light smooth ring D , round pegs at B and C , again through the ring, and is attached to a weight at its free extremity E . Prove that in the position of equilibrium the ring and the weight hang vertically below B .

3. Show that if three forces in one plane be in equilibrium, their lines of action must be parallel or meet in a point.

A uniform triangular lamina ABC of weight W lying on a smooth horizontal table is just raised by a vertical force applied at A ; find the magnitude of this force and that of the resultant pressure between the base BC and the table. Show that if the applied force be not vertical the base BC will not remain at rest.

4. Define the "moment of a force about a point," and explain how the sign of the moment is to be determined.

The sides BC, CA, AB of a triangle are 3, 4 and 5 feet respectively; find the magnitude and direction of a force acting at C whose moments about A and B are 7 and 5 foot lbs. respectively.

5. Find the relation between the power and the weight in the system of pulleys in which the same string passes round all the pulleys (neglecting the weight of the pulleys).

Draw diagrams of the above system in which weights of 6 and 7 lbs. are balanced by downward forces of 1 lb.

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6. Define the "centre of gravity" of a body, and show that if a body be freely suspended from a point O , its centre of gravity will be in the vertical line through O in the position of rest.

A uniform wire is bent into the form of an equilateral triangle, how must it be suspended in order that it may hang with one side vertical?

7. Define "velocity" and "acceleration," and explain how each is measured when uniform.

Compare the velocities of two points one of which describes 2 miles in 7 minutes and the other 60 yards in 11 seconds.

8. Explain how Attwood's machine may be used to determine approximately the measure of the acceleration due to gravity.

9. A body starts from rest subject to a uniform acceleration f . Write down any two relations between s the space described, t the time, v the velocity acquired, and f .

A particle is let fall from a point 91 feet above the ground. After describing 36 feet it meets with an obstruction which diminishes its velocity by one-half. Find the whole time of motion and also the velocity the particle has when it strikes the ground.

10. Explain how force is measured in Dynamics, and obtain the value of the British unit of force.

What force must act upon a mass of 65 lbs. to increase its velocity from 32 to 33 feet per second in passing over 50 feet? State clearly in terms of what unit your answer is expressed.

11. Enunciate the second law of motion and show how it may be applied to investigate the path of a projectile in vacuo.

A particle is projected from a point O and two lines OA , OB are drawn to represent in magnitude and direction the velocity of projection and that due to gravity at the end of one second. Construct geometrically the positions of the particle at the end of the first, second, and third seconds of its motion.

FURTHER EXAMINATION.

V. MATHEMATICS (1).

*[Full marks may be obtained for about three-fourths of this paper.
Great importance is attached to accuracy.]*

1. Distinguish between the different sections of a cone. What do central conics become (i.) when equal diameters are at right angles, (ii.) when a vertex is at the centre or at infinity?

2. If the normal at any point P of a conic meet the axis in G , show that

$$SG : SP :: SA : AX.$$

Show that as P approaches A , PG tends to become equal to the semi latus-rectum.

3. Prove that the tangent at any point of a parabola bisects the angle between the focal distance and the diameter produced.

If Y, Y' be the feet of the perpendiculars from the focus on the tangent and normal at any point, show that YY' is parallel to the axis of the parabola.

4. If CP, CD be conjugate radii of an ellipse, show that

$$(i.) \quad SP \cdot S'P = CD^2;$$

$$(ii.) \quad (AC - SP)^2 + (SD - AC)^2 = CS^2.$$

5. Define an asymptote, and show that each asymptote of a hyperbola is a diameter of the conic which is conjugate to itself.

If two hyperbolas have the same asymptotes, prove that any chord of one which touches the other is bisected at the point of contact.

6. If two spheres be inscribed in a cone so as to touch the same plane section of the cone, show that they touch the section at its foci.

Show that the locus of the foci of all the parabolic sections of a cone, whose vertical angle is 2α , is another cone (vertical angle 2θ), such that

$$\tan(\theta + \alpha) = 2 \tan \alpha.$$

7. Draw the lines and curves represented by the following equations:

(i.) $x^2 - y^2 - 6y - 9 = 0$;

(ii.) $x^2 + y^2 - 3x - 4y = 0$;

(iii.) $y^2 - 4ay - 4ax = 0$;

(iv.) $x^2 + 4y^2 - 12x - 32y = 0$.

8. Find, referred to rectangular coordinates, the equation to the straight line which passes through (x', y') , and makes an angle α with the axis of x .

Straight lines, all passing through a fixed point, cut two fixed straight lines which are at right angles; show that the locus of the middle point of the intercept between the fixed lines is a hyperbola; and find its asymptotes, and draw the curve.

9. Write down the general equation to a circle in rectangular coordinates; and show from it that if a straight line OPQ be drawn from the origin O to meet the circle in P and Q , the rectangle $OP \cdot OQ$ is constant for all directions of OPQ .

10. Show that $y = mx - 2am - am^3$, is the equation to a normal to $y^2 = 4ax$.

Find the equation to the locus of the fourth vertex of the rectangle, of which three vertices are the vertex of the parabola and the intersections of any normal with the axis and the tangent at the vertex.

11. Find the equation to a chord of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

in terms of the eccentric angles of the extremities of the chord.

If the difference between the eccentric angles be 60° , show that the intercepts (α and β) of the chord on the axes of the ellipse are connected by the equation

$$\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = \frac{4}{3}.$$

12. Prove that

$$\frac{l}{r} = 1 + e \cos \theta \quad \text{and} \quad \frac{l}{r} = \cos(\theta - \alpha) + e \cos \theta$$

are the polar equations respectively of a conic, and of a straight line which only meets the conic in one point.

Show that the line joining the focus to any external point bisects the angle between the focal vectors to the extremities of the chord which is the polar of the point.

VI. MATHEMATICS (2).

[Full marks may be obtained for about three-fourths of this paper. Great importance is attached to accuracy.]

1. If two arithmetic means are inserted between two given quantities, show that the product of the means is greater than the product of the given quantities.

2. If any finite values be assigned to x, y, z so as to satisfy the indeterminate equations

$$x + \frac{a^2}{x} = y + \frac{a^2}{y} = z + \frac{a^2}{z},$$

show that two, at least, of the values must be equal, and obtain a solution in which they are not all equal.

3. Expand in a series of ascending powers of x , each of the expressions

$$(1 - x + x^2)^{-1}, \quad (1 - x - 2x^2)^{-1}.$$

If p_n, q_n be the respective coefficients of x^n in the two series, show that $3q_n - p_{3n} = 2^{n+1}$.

4. Prove that if the two continued fractions

$$\frac{1}{a + \frac{1}{a - \frac{1}{a + \frac{1}{a - \frac{1}{a + \dots}}}}}, \quad \frac{1}{a - \frac{1}{a + \frac{1}{a - \frac{1}{a + \frac{1}{a - \frac{1}{a + \dots}}}}},$$

be continued indefinitely, their sum is $\frac{2}{a}$.

5. Explain what is meant by the "scale of relation" of a "recurring series."

Find the scale of relation and the n^{th} term of the series

$$1 + 5 + 7 + 17 + 31 + \dots$$

6. Find the number of cannon balls in a pyramidal pile, having 10 balls in each of the four sides of its base.

Also find the vertical height of the single ball at the top above the horizontal plane on which the pile rests in terms of the diameter of each ball.

7. If $\tan 4\theta = 3 \tan 2\theta$, show that $\sin 6\theta = 2 \sin 2\theta$, and find all the values of θ which satisfy these equations.

8. Find the length of the straight line drawn bisecting one angle of a triangle and terminated by the opposite side.

If d_1, d_2, d_3 be the lengths of the lines so drawn from the angular points A, B, C respectively, and r be the radius of the circle inscribed in the triangle ABC , prove that

$$\frac{\operatorname{cosec} \frac{A}{2}}{d_1} + \frac{\operatorname{cosec} \frac{B}{2}}{d_2} + \frac{\operatorname{cosec} \frac{C}{2}}{d_3} = \frac{2}{r}.$$

9. Write down the expansion of the cosine of an angle in a series of powers of its circular measure, and deduce the exponential value of the cosine.

Show that the five roots of the equation $x^5 = -1$, form a geometrical progression whose ratio is $e^{\frac{2\pi\sqrt{-1}}{5}}$.

10. A man walking along a straight road notes when he is in the line of a long straight fence, and observes that 78 yards from this point the fence subtends an angle of 60° , and that 260 yards further on this angle is increased to 120° . When he has walked 260 yards still further, he finds that the fence again subtends an angle of 60° . Find the angle which the direction of the fence makes with the road, and show that its middle point is 120 yards distant from the road.

11. Sum to $n-1$ terms each of the series

$$(1) \quad \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \sin \frac{3\pi}{n} + \dots,$$

$$(2) \quad x \sin \frac{\pi}{n} + x^2 \sin \frac{2\pi}{n} + x^3 \sin \frac{3\pi}{n} + \dots$$

VII. MECHANICS.

[Full marks may be obtained for about three-fourths of the paper. Mathematical instruments must be used for questions requiring a graphical method of solution.]

1. Find the resultant of two unlike parallel forces acting upon a rigid body.

AB , CD represent two unlike parallel forces acting at the points A , C . Find graphically the position of the line of action of their resultant, and also the magnitude of this resultant if one foot represent a force of 50 lbs.

$$AB = 1.18 \text{ in.}, \quad CD = 2.16 \text{ in.}, \quad AC = 1.38 \text{ in.}, \quad \angle BAC = 69^\circ.$$

2. If four forces in one plane be in equilibrium, and the lines of action of all be given, but the magnitude of only one, show how the magnitudes of the other three may *generally* be determined by the graphical method.

Mention the cases in which the problem is indeterminate, and show how the method of construction fails. Apply the method to the graphical solution of the following problem.

The lower extremity of a uniform beam ED , length 5.1 in., rests on the ground at the foot of a vertical wall EF , its upper extremity being attached by a cord DF , length 2.6 in., to a point F 5.5 in. vertically above E . The weight of the beam being 200 lbs., find the tension of the cord and the pressures of the beam against the wall and ground at E .

3. Prove that, if a system of forces acting in one plane upon a particle be in equilibrium, the algebraical sums of their resolved parts in two directions at right angles are respectively equal to zero.

A small heavy ring of weight W can slide upon a smooth wire in the form of a parabola whose equation is $y^2 = 4ax$, the axes of x and y being respectively horizontal and vertical. A string attached to the ring passes over a smooth peg at the focus and supports a weight P at its other extremity. Find the coordinates of the ring in the position of equilibrium and show that the pressure between the wire and the ring = $\frac{P^2 \sim W^2}{\sqrt{P^2 + W^2}}$.

4. State any three necessary and sufficient conditions that a system of forces in one plane may be in equilibrium.

Three forces, P_1, P_2, P_3 , act at the middle points of the sides a, b, c of a triangle ABC , and at right angles to those sides. Show that if they form a system in equilibrium with Q_1, Q_2, Q_3 acting along the same sides, then (the forces being taken in the proper directions)

$$Q_1 \times 4 \text{ area of triangle} = (P_2a - P_1b)b - (P_3a - P_1c)c$$

and two similar equations.

5. Find the centre of gravity of a quadrilateral figure which has two sides parallel.

The lengths of the two parallel sides AB, CD of a quadrilateral lamina $ABCD$ are $a, 2a$ respectively, and the weight is W . If the lamina be supported in a horizontal position at the points B, C, D , find the pressures on the supports in terms of W .

6. Investigate the relation between the power and the weight in the case of the smooth screw.

7. Define work, and show that, if a number of weights be raised through different heights, the amount of work done is equal to that in raising a weight equal to the sum of the weights through the height through which the centre of gravity of the weights has been lifted.

An iron cylinder, 12 feet long, 5 feet in diameter, and weighing 1000 cwt., is lying on the ground: find the least work that must be expended to raise it into a vertical position.

8. Prove the formula $s = ut + \frac{1}{2}ft^2$, and deduce an expression for the space described in the n^{th} second.

If t represent the whole time of motion to be considered, and if a be the space described in the first m seconds, and b the space described in the last m seconds, show that $s = \frac{(a+b)t}{2m}$.

9. A mass P hanging freely, draws another mass Q along a smooth horizontal table by means of a string passing over a pulley at its edge: find the acceleration (f) and the tension of the string (T).

If the table be rough, show that the acceleration will be diminished by $\mu(g-f)$, where μ is the coefficient of limiting friction.

10. Two elastic spheres, equal in all respects, are moving towards each other with equal velocities, their centres being on two parallel lines whose distance apart is d_1 (less than d the diameter of either sphere). Prove that after impact they will move away from each other with equal velocities so that their centres are on two parallel lines whose distance apart d_2 is given by the equation

$$d_2^2 \{ e^2 d^2 + (1 - e^2) d_1^2 \} = d^2 d_1^2.$$

11. A particle is placed upon a rough horizontal plate ($\mu = \frac{2}{3}$) at a distance of 9 inches from a vertical axis about which the plate can turn : find the greatest number of revolutions per minute the plate can make without causing the particle to move upon it.

12. Find the time of oscillation of a simple pendulum of given length.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1891.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.]

1. Draw a straight line perpendicular to a given straight line of an unlimited length, from a given point without it.

2. Describe a parallelogram which shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

3. On the perpendicular AD of an equilateral triangle ABC another equilateral triangle EAD is described; show that its perpendicular EF is one-fourth of the perimeter of the triangle ABC .

4. Enunciate that proposition in Euclid's second book which is expressed directly in algebraic symbols by the formula $(2a+b)b+a^2=(a+b)^2$, and give the construction by which the proposition is proved.

5. In any triangle the square on the side subtending an acute angle is less than the squares on the sides containing that angle by twice the rectangle contained by either of those sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle and the acute angle.

A, B are two given points, and CD a given straight line ; in CD find a point P such that the difference between the squares on AP, BP shall be equal to twice the square on AB .

6. Make a square equal in area to a given equilateral triangle.

7. Draw a tangent to a given circle from a given point outside it.

8. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

If the opposite sides of such a quadrilateral be produced to meet in P and Q , and about the triangles so formed outside the quadrilateral circles be described intersecting again in R , show that P, R, Q will be in one straight line.

9. Describe a circle which shall touch one side of a given triangle and the other two sides produced.

10. Describe an isosceles triangle having each angle at the base double of the vertical angle.

Construct a quadrilateral whose angles, taken in order, are in the proportion $1 : 2 : 3 : 4$, and whose longest side is double of that opposite to it.

11. If the vertical angle of a triangle be bisected by a straight line which cuts the base, the segments of the base shall have the same ratio which the other sides of the triangle have to one another.

The angle A of a triangle ABC is bisected by AD , which cuts the base at D , and O is the middle point of BC ; show that OD bears the same ratio to OB that the difference of the sides bears to their sum.

12. In any right-angled triangle, any rectilineal figure described on the side subtending the right angle is equal to the similar and similarly-described figures on the sides containing the right angle.

II. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—Great importance will be attached to accuracy.]

1. Prove that

$$(4x-1)^3 + (2x-3)^3 + 6(4x-1)(2x-3)(3x-2) = 8(3x-2)^3;$$

and divide :

$$(x-1)^8 + (x-1)^4 + 1 \text{ by } (x-1)^4 + (x-1)^2 + 1.$$

2. Simplify

$$\frac{2(x^2-1)^2}{(x^2+2)(x^2-16)} - \frac{(x-4)(x-1)^2 + (x+4)(x+1)^2}{2x(x^2-16)};$$

and resolve into factors each of the expressions

$$x^2 - 26x + 168 \text{ and } x^4 + x^2y^2 + y^4.$$

3. Show that the sum of the squares of 5 consecutive numbers is divisible by 5.

4. Find the highest common factor of

$$4x^4 + 7x^2 + 16 \text{ and } 4x^4 + 12x^3 + 9x^2 - 16.$$

5. Solve the equations

$$(i.) \quad \frac{3}{x} + \frac{5}{x^2} = \frac{3}{x-1} + \frac{2}{(x-1)^2};$$

$$(ii.) \quad \begin{cases} \frac{16x^2}{9} - \frac{36y^2}{25} = 3, \\ \frac{4x}{3} - \frac{6y}{5} = 1. \end{cases}$$

6. Transform 5363433 from the septenary scale to the denary.

Multiply together the two numbers 4535 and 246 in the septenary scale.

7. Solve the equations

$$(i.) \quad \frac{ax}{b} - \frac{b}{ax} = b - \frac{1}{b};$$

$$(ii.) \quad \begin{cases} x^3 - xy + y^3 = 4, \\ x + y = 1. \end{cases}$$

WOOLWICH ENTRANCE EXAMINATION.

8. A courtyard, containing 1300 square feet, is paved with 50 stones of one size and 195 of another. If 10 stones of the first size and 7 of the second make together 100 square feet, find the size of each stone.

9. By *Mathematical Induction*,

- (i.) prove the formula for the summation of a Geometrical Progression ;
(ii.) show that the series

$$(1^5 + 1^7) + (2^5 + 2^7) + (3^5 + 3^7) + \text{etc. (to } n \text{ terms)} = \frac{n^4(n+1)^4}{8}.$$

10. Find an infinite geometrical progression whose first term is 1, and in which each term is twice the sum of all the terms that follow it.

Define *Harmonical Progression*; and place two terms at each end of the H.P. 15, 20, 30.

11. Find for what value of r the number of combinations of $2m$ things taken r at a time is greatest.

Show that there are two terms in the expansion of $(1-x)^{-\frac{5}{3}}$ greater than all the others, when $x = \frac{1}{17}$.

12. When the logarithms of two adjacent numbers are known, explain how to determine the logarithm of a number lying between them.

Find the tenth root of 4325626.

If £35627. 19s. 4d. be placed at compound interest at the rate of $3\frac{1}{2}$ per cent. per annum, what will be the amount at the end of 30 years?

III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy.]

1. The tangent of an angle is 2.4. Find the values of the cosecant of the angle, the cosecant of the complement of half the angle, and the cosecant of the supplement of double the angle.

2. Establish the identities—

$$(i.) (\operatorname{cosec} A + \sec A)^2 - (\cos A + \sin A)^2 \\ = (\cot A \operatorname{cosec} A + \tan A \sec A)(\sin A + \cos A)(1 + \sin A \cos A),$$

$$(ii.) 2 \frac{\sin^2 A}{1 + \cot A} + \sin 2A + \frac{2 \cos^2 A}{1 + \tan A} = 2.$$

3. Simplify

$$\frac{\cos(n-2)A - \cos nA}{\sin nA - \sin(n-2)A};$$

and prove that

$$4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239} = \frac{\pi}{4}.$$

4. Prove that

$$\sqrt{1 + \tan \theta + \tan^2 \theta} \sqrt{1 + \cot \theta + \cot^2 \theta} - \sec \theta \operatorname{cosec} \theta = 1.$$

5. In a plane triangle show that the sines of the angles are to one another as the opposite sides.

Show that

$$a \cos A \cos 2A \sin 4(B-C) + b \cos B \cos 2B \sin 4(C-A) \\ + c \cos C \cos 2C \sin 4(A-B) = 0.$$

6. In the ambiguous case in the solution of plane triangles, if a and b are the two given sides $a < b$, A the given angle, and C_1, C_2 the two values of the angles opposite the third side, prove that

$$\cos A \cos B = \sin \frac{C_1 + C_2}{2} \cdot \sin \frac{C_1 - C_2}{2},$$

taking the smaller of the two values of B .

7. In any triangle ABC , if r, r_a, r_b, r_c , are the radii of the inscribed and of the three escribed circles respectively, and p_a, p_b, p_c , the perpendiculars from the angles on the sides a, b, c , prove that

$$\frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c} = \frac{1}{r} = \frac{1}{p_a} + \frac{1}{p_b} + \frac{1}{p_c};$$

and if the points where the perpendiculars meet the sides be joined, the perimeter of the triangle so formed is $2a \sin B \sin C$.

8. Find the values of the angle C in the triangle ABC from the following data :

$$b = 785, \quad c = 1083, \quad B = 37^\circ 11' 45''.$$

9. Adapt to logarithmic computation the expression

$$(x + \sqrt{x^2 - 1})^{\frac{1}{2}} + (x - \sqrt{x^2 - 1})^{\frac{1}{2}}.$$

10. A railway tunnel consists of a hollow semi-cylindrical top, terminated below in a trough with slanting sides and flat base. The radius of the former being 12 feet, the base and height of the latter being 20 and 18 feet respectively, and the length of the tunnel 1200 yards, find the cost of facing the sides and roof with brick at 1s. 6d. per square foot ($\pi = \frac{22}{7}$).

11. Within a hollow sphere of 1 foot radius is placed a right prism, the ends of which are equilateral triangles. The side of one of these being 1 foot in length, and the surface of the sphere being in contact with all the six angular points of the prism, find in cubic inches the volume of the latter.

IV. STATICS AND DYNAMICS.

1. Enunciate and prove the "Triangle of Forces."

A small heavy ring, which can slide freely upon a smooth thin rod, is attached to the end of the rod by a fine string. If the rod be held in any position inclined to the vertical, draw a triangle representing the forces acting upon the ring.

2. A rectangular box, containing a ball of weight W , stands on a horizontal table, and is tilted about one of its lower edges through an angle of 30° . Find the pressures between the ball and the box.

3. State the conditions of equilibrium of three parallel forces acting upon a rigid body.

A rod ABC , 16 inches long, rests in a horizontal position upon two supports at A and B one foot apart, and it is found that the least upward and downward forces applied at C which would move the rod are 4 oz. and 5 oz. respectively. Find the weight of the rod and the position of its centre of gravity.

4. Show how to find the centre of gravity of a body composed of two parts whose weights and centres of gravity are known.

A solid figure is formed of an upright triangular prism surmounted by a pyramid; if the length of every edge of this figure be a feet, find the height of its centre of gravity above the base.

5. Show that the algebraical sum of the moments of two forces (whose lines of action intersect) about any point in the plane containing the forces is equal to the moment of their resultant.

OA , OB are chords, 4 and 5 inches in length, of a circular disk $OACB$, whose diameter OC is 6 inches. If forces of 3 and 4 lbs. act from O along these chords respectively, find how the disk will begin to move, the point C being fixed.

6. Describe, and show how to graduate, the Danish Steelyard.

7. Explain how the measure of the velocity of a moving point depends upon the units of space and time.

If the unit of time be half a minute and the unit of length be 2 yards, what will be the measure of the velocity of a body which describes, at a uniform rate, 14 miles in 3 hours?

8. Investigate the formula $s = \frac{1}{2}ft^2$ for the space described from rest by a particle subject to uniform acceleration f , and hence deduce a corresponding expression in the case where the particle has an initial velocity u .

A particle is observed to describe 7 feet in 3 seconds, and 13 feet in the next 3 seconds; find its acceleration.

9. The side BC of a triangle ABC is vertical; show that, if the times of falling down the two sides BA , AC be equal, the triangle must be isosceles or right angled.

10. Distinguish, with examples, between the *volume*, *mass*, and *weight* of a body, and find the relation between the units of mass and weight in order that W may be equal to Mg .

If the measures of the mass and weight be the same, and the unit of length be 2 feet, find the unit of time.

11. If a force of 15 poundals act upon a mass of 13 pounds, what velocity will it generate in 8 seconds?

12. A particle is projected in any manner in a vertical plane, show how to find its position at the end of a given time.

A stone is thrown from the top of a tower with a velocity of 50 feet a second in a direction making an angle of 30° with the horizon. Find the distance of the stone from the point of projection at the end of 5 seconds.

FURTHER EXAMINATION.

V. MATHEMATICS (I).

[Full marks may be obtained for about three-fourths of this paper. Great importance is attached to accuracy.]

1. Define a "conic section," and show that a straight line *generally* cuts a conic section in two points.

Enumerate carefully all the cases of exception.

2. Prove that the subtangent at any point of a parabola is double of the abscissa, and the subnormal is half of the latus rectum.

If one side of an equilateral triangle be a focal radius SP of a parabola, and another side lie along the axis, show that the third side will be either the tangent or the normal at P , and find in each case the length of SP .

3. Prove that, in any central conic, the square on the ordinate PV bears a constant ratio to the rectangle under the abscissae $AN, A'N$.

PNP' is a double ordinate of an equilateral hyperbola whose transverse axis is AA' . Prove that the directions of $AP, A'P'$ are perpendicular to one another.

4. If from any point O two straight lines be drawn, $OPQ, OP'Q'$, intersecting an ellipse in $P, Q; P', Q'$, show that the rectangles $OP \cdot OQ, OP' \cdot OQ'$ are to one another as the squares of the diameters parallel to $PQ, P'Q'$ respectively.

If an ellipse, inscribed in a triangle, touch one side in its middle point, prove that the straight line joining that point with the opposite angle of the triangle will pass through the centre of the ellipse.

5. If from any point P on a hyperbola a straight line, drawn in a given direction, meet the two asymptotes in R, R' respectively, prove that the rectangle $RP \cdot PR'$ is constant.

Hence show that a straight line touching the hyperbola, and terminated by the asymptotes, is bisected at the point of contact.

6. If a right cone be cut by a plane, determine by geometrical construction the positions of the foci and directrices of the curve of section.

Show that sections made by parallel planes have the same eccentricity.

7. Given the coordinates of two points, P, Q , find the coordinates of the point in which the straight line PQ is divided in a given ratio.

Apply your results to show that the straight line joining the middle points of the diagonals of a quadrilateral, and the straight line joining the middle points of either pair of opposite sides, bisect each other.

8. Find the angle between the straight lines whose equations, referred to rectangular axes, are

$$lx + my + c = 0, \quad l'x + m'y + c' = 0.$$

$OACB$ is a parallelogram, whose sides OA, OB are of lengths a, b , and make angles α, β respectively with the axis Ox . Write down the equations of the two diagonals OC, AB , and show that the tangent of the angle between them is

$$\frac{2ab \sin(\beta - \alpha)}{a^2 - b^2}.$$

9. Prove that the equation of any tangent to the circle $x^2 + y^2 = c^2$ may be written in the form

$$x \cos \alpha + y \sin \alpha = c.$$

Prove also that the locus of the foot of the perpendicular let fall upon the tangent, from the point in which the circle cuts the positive direction of the axis of y , is the curve whose equation is

$$(x^2 + y^2 - yc)^2 = c^2 \{x^2 + (y - c)^2\}.$$

10. Show that the equation of the normal to the parabola $y^2 = 4ax$, which is inclined at an angle θ to the axis, may be written in the form

$$y = (x - a - a \sec^2 \theta) \tan \theta.$$

If a chord of a parabola, whose inclination to the axis is $\tan^{-1} \sqrt{2}$, be normal to the curve at one end, show that it will subtend a right angle at the vertex.

11. Investigate the equation of the polar of any point with respect to a given ellipse or hyperbola.

An ellipse and a hyperbola have the same principal axes. Show that the polar of a point on either curve with respect to the other touches the first curve.

12. Write down the equations of the latera recta of the conic whose polar equation is

$$\frac{L}{r} = 1 - e \cos \theta.$$

Also show that if r_1, r_2, r_3 be the lengths of three focal radii, each of which is equally inclined to the other two,

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{3}{L}.$$

VI. MATHEMATICS (2).

[Full marks may be obtained for about three-fourths of this paper. Great importance is attached to accuracy.]

1. If $a^3 + b^3 + c^3 = 3abc$, where a, b, c are finite, show that any one of the three quantities

$$a^2 - bc, \quad b^2 - ca, \quad c^2 - ab,$$

is a mean proportional between the other two.

2. Find the coefficient of x^6 in the expansion of

$$(1 + x^2 + x^4)(2 - 3x - 2x^2)^{-1}$$

in a series of powers of x , and show that

$$\sqrt[5]{.875} = .97365 \text{ nearly.}$$

3. A bag contains 3 balls, which are equally likely to be either white, black, or red. A white ball is drawn and replaced, and then a black ball is drawn and replaced. Find the probability that the next drawing will give a red ball.

4. Express

$$\sqrt{a^2 + \frac{2a}{2a+1}}$$

as a continued fraction, and show that

$$1 + \frac{1}{2+} \frac{1}{6+} \frac{1}{2+} \frac{1}{6+} \dots$$

is double of

$$\frac{1}{1+} \frac{1}{2+} \frac{1}{1+} \frac{1}{2+} \dots$$

5. In how many different ways can 3 white, 3 red, and 3 blue counters be arranged in 3 rows of 3 each?

If there were n counters of each of n different colours, show that the number of ways in which the n^2 counters could be arranged in n rows of n each is

$$\frac{n^2}{1^n \times 2^n \times 3^n \times \dots \times n^n}$$

6. Solve the simultaneous equations

$$(i.) \quad \begin{cases} x^2 + y^2 = 2a^2 \\ x^3 - y^3 = 2a^3. \end{cases}$$

$$(ii.) \quad \begin{cases} (x \cos \alpha - c) \sin(\alpha + \beta) = (y \cos \alpha - c) \sin(\alpha - \beta) \\ x \cos(\alpha + \beta) = y \cos(\alpha - \beta). \end{cases}$$

Show that in (ii.), $x + y = 2c \cos \alpha$.

7. If θ be the circular measure of an angle less than a right angle, prove that

$$\sin \theta > \theta - \frac{\theta^3}{4}, \text{ and } \tan \theta > \theta + \frac{\theta^3}{4}.$$

8. A man, travelling due north along a straight road, observes the points A, B at which two distant objects P, Q lie respectively due east, and at an intermediate point C he measures the distances AC, BC , and the angles PCA, QCB . Show that he will have data from which he can calculate the length of PQ and its inclination to the direction of the road.

Show that this inclination will be 30° , if each of the measured angles is 60° , and one of the measured distances is double of the other.

9. If $2 \cos \theta = x + \frac{1}{x}$, express $\cos n\theta$ and $\sin n\theta$ in terms of x .

Also prove that

$$\frac{\tan^{-1}(\sqrt{-1} \sin \theta)}{\sqrt{-1}} = \log \tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right).$$

10. An open box in the shape of a perfect cube lies on a horizontal plane with the diagonals of its base pointing towards the four cardinal points. Show that at noon, when the sun's altitude is $\tan^{-1}(2\sqrt{2})$, the portions of the interior surface in sunlight and in shade are as 37 : 43.

11. Find the sum of each of the infinite series

$$(i.) \quad \frac{1}{2 \cdot 3} + \frac{1}{4 \cdot 5} + \frac{1}{6 \cdot 7} + \dots;$$

$$(ii.) \quad \frac{1}{3 \cdot 5} + \frac{1}{7 \cdot 9} + \frac{1}{11 \cdot 13} + \dots;$$

$$(iii.) \quad \frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \dots.$$

VII. MECHANICS.

[Full marks may be obtained for about three-fourths of the paper. Mathematical instruments must be used for questions requiring a graphical method of solution.]

1. Solve graphically the following problem :—A uniform beam AB , weight 100 lbs., is supported by two strings AC , BD , the former making an angle of 51° with the horizontal, and the latter being vertical. It is maintained at an angle of 5° to the horizon by a horizontal force P applied at B . Find, by graphical construction, the value of P in pounds.

2. Solve the following problem :—A uniform horizontal beam AB , weight 100 lbs., is placed with the end A resting against a rough wall AD , and is supported by a string CD . Find, by construction, the tension of CD in pounds, and the coefficient of friction in operation between the beam and the wall at A .

$$AB = 4', \quad AC = 3', \quad AD = 2'.9''.$$

3. Three forces, P , Q , R , make equilibrium ; P is given in magnitude and position, Q in magnitude only, R in direction only, making an angle θ with P . Determine Q and R either by construction or by calculation. Show that there are generally two solutions, and that if $P > Q$ there are limits to the angle θ , beyond which the question is impossible of solution.

4. If four forces in equilibrium act along the sides of a parallelogram, show that they are as the sides in which they act.

If four forces in equilibrium act at right angles to the sides of a parallelogram (not rectangular) at their middle points, prove that they are as the sides on which they act.

5. A rectangular block $ABCD$, whose height is double its base, stands with its base AD on a rough floor, coefficient of friction $\frac{1}{2}$. If it be pulled

by a horizontal force at C till motion ensues, determine whether it will slip on the floor, or begin to turn over round D .

6. The masses of two particles at A, B are m, m' : if P be any point, prove that the resultant of two forces represented by $m \cdot AP, m' \cdot BP$ acts in GP , and is represented by $(m+m')GP$, where G is the centre of gravity of m, m' . Hence show that if G is the centre of gravity of the masses m, m', m'' at any three points A, B, C , the three forces

$$m \cdot AG, m' \cdot BG, m'' \cdot CG$$

make equilibrium.

If OA, OB, OC are three diverging bars of the same material and same section, prove that if their centre of gravity is at O , the sines of the angles they make are as the squares of their lengths.

7. Find the slope of a smooth inclined plane, if the work done in drawing a heavy body up a given length of it, is equal to that done in drawing the body along an equal length of a rough floor (coefficient of friction $\frac{1}{2}$).

If the floor and plane are equally rough ($\mu < 1$), prove that more work is done in drawing it up the plane, than along an equal length of the floor, whatever be the slope.

8. If a body is projected vertically upwards, prove that the velocity at any point in its ascent is equal to its velocity when passing through that point in its fall.

9. Find the amount of *vis viva* lost in the direct collision of two inelastic balls, masses M, m , velocities u, v .

If the balls are equal and going in the same direction, show that less than half their *vis viva* is lost.

10. A flexible heavy string, length $2l$, is moving over a smooth fixed small pulley, the two unequal portions of it hanging vertically. Prove that at the instant when its middle point is at a distance x below the pulley, the acceleration with which it is moving is

$$f = \frac{x}{l}g.$$

Find also the tension of the string at any assigned point of the descending portion at the same instant.

WOOLWICH ENTRANCE EXAMINATION.

11. A projectile thrown at a small elevation (3°), gives a range of 1000 yards on a horizontal plane. If the plane, instead of being horizontal, had an upward slope of 1° , what would be the range in yards, approximately?

12. A heavy bead, loosely strung on a smooth vertical circular wire, falls down it from rest at the highest point O . When at any assigned point, find the velocity with which its distance from O (in a straight line), is increasing.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1891.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.]

1. Draw a straight line at right angles to a given straight line, from a given point in the same; and show that two straight lines cannot have a common segment.

2. Give Euclid's Axiom on parallel straight lines; and show that the straight lines which join the extremities of two equal and parallel straight lines towards the same parts are also themselves equal and parallel.

Two quadrilaterals $ABCD$, $EFGH$, have the sides AB , DC respectively equal to the sides EF , HG ; and also the angles which the diagonals AC , BD make with AB , CD , namely, CAB , ABD , BDC , DCA respectively equal to the angles which the diagonals EG , FH make with EF , HG , namely, GEF , EFH , FHG , HGE ; show that the quadrilaterals are equal in every respect.

3. To a given straight line apply a parallelogram, which shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

4. Enunciate the two propositions represented by

$$(2a \pm b)^2 + b^2 = 2\{a^2 + (a \pm b)^2\}.$$

AB is bisected at *C* and produced to *D*; *AE*, which is at right angles to *AB*, is divided into two equal parts at *F* and into two unequal parts at *G*; *CH*, *BI*, *DKL* are drawn parallel to *AE*; and *FK*, *GHL*, *EI* parallel to *AD*. Show that the sum of the squares on *GD* and *IL* is double of the sum of the squares on *FC* and *HK*.

5. In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

6. From any point in the diameter of a circle which is not the centre there can be drawn to the circumference two straight lines, and only two, which are equal to one another, one on each side of the diameter.

If two such pairs of equal straight lines are drawn, prove that the chords joining the extremities of the unequal straight lines meet upon the diameter of the circle (produced if necessary).

7. If from any point without a circle two straight lines be drawn, one of which cuts the circle, and the other touches it; the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, shall be equal to the square on the line which touches it.

8. A quadrilateral *ABCD* is circumscribed round a circle, touching the circle in *E*, *F*, *G*, *H*. Find the difference of two opposite angles of *ABCD* in terms of the difference of two adjacent angles of *EFGH*.

9. Describe a circle about a given triangle.

10. Inscribe a regular quindecagon in a given circle.

11. The sides about the equal angles of equiangular triangles are proportionals.

If in two triangles *ABC*, *DEF*, *AB* is to *BC* as *DE* is to *EF*, and the angle *A* is equal to the angle *D*, but the angle *B* unequal to the angle *E*; show that the angles *C* and *F* are supplementary.

12. In equal circles angles, whether at the centres or at the circumferences, have the same ratio which the arcs on which they stand have to one another.

II. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—Great importance will be attached to accuracy.]

1. Find the value of

$$1680 + (r-7)[1470 + (r-8)\{378 + (r-9)(35 + r-10)\}]$$

when $r = -7$.

2. Multiply

$$2x^5 + 4x^4 + 6x^3 + 8x^2 + 10x + 5 \text{ by } x^2 - 2x + 1.$$

Find an algebraic function such that when it is divided by $a^2 - ab + b^2$, the quotient is $3a^2 - 2ab + b^2$ and the remainder is $2a^3(b-a)$.

3. Write down one factor of $x^{17} - 17x^2 + 16$.

Divide $x^{17} - 17x^2 + 16$ by $x^2 - 2x + 1$, so far as to show the four terms of highest degree in the quotient.

4. Define the algebraical Greatest Common Measure and Least Common Multiple of two or more functions.

Find the Greatest Common Measure of

$$7x^4 + 6x^3 - 8x^2 - 6x + 1 \text{ and } 11x^4 + 15x^3 - 2x^2 - 5x + 1.$$

5. Simplify the expression

$$\frac{(x-y)^3 + (z-y)^3 - (x-2y+z)^3}{(x-y)(z-y)(x-2y+z)}.$$

6. Solve the equations

$$(i.) \quad \frac{1}{x+2} + \frac{1}{x+3} - \frac{3(x+6)}{(x+2)(x+3)} = 6 - \frac{6x+23}{x+3};$$

$$(ii.) \quad \begin{aligned} 11x + 4y &= 71, \\ 11y + 4x &= 51. \end{aligned}$$

7. Examine the meaning of $a^{\frac{1}{2}}$ and a^0 in the theory of indices.

If
show that

$$\begin{aligned} 3c &= \sqrt[3]{100} + 2\sqrt[3]{10} + 3, \\ 3(c-1)^3 - 20c &= 0. \end{aligned}$$

WOOLWICH ENTRANCE EXAMINATION.

8. Solve the equations

(i.) $x^2 - 18x - 72 = 0$;

(ii.) $\sqrt{3}\sqrt{(x^4 + 2x^2 + 3)} + \frac{1}{3}x^2 + 3 = \frac{2}{3}ax^2$.

9. Find the sum of 19 terms of the Arithmetical Progression whose 3rd and 13th terms are 17 and 87 respectively.

If $x!$ denote the product of the first x integers, show that

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + x \cdot x! = (x+1)! - 1.$$

10. Find the number of permutations of n different things taken 4 together.

If there are 4 flags of each of 6 different colours, prove that 4 flags may be hoisted into a vertical line in 6^4 different ways.

11. Expand $\frac{1}{(1-x)^n}$ by the binomial theorem, stating the necessary limitations on the value of x .

If

$$(1-x)^{-1}(1-x^2)^{-p}(1-x^3)^{-q}(1-x^4)^{-r} = 1 + x + 2(px^2 + qx^3 + rx^4) + \dots$$

show that

$$p = \frac{1}{2}q = \frac{1}{3}r = 1.$$

12. If a number has no integral part, show that the characteristic of its logarithm to base 10 is negative, and numerically exceeds by unity the number of zeros following the decimal point.

Calculate

$$(2.718)^{3.142} \times (3.142)^{2.718}$$

by the aid of a table of logarithms.

III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy.]

1. Trace the changes in sign and magnitude

$$(1) \text{ of } \sin \theta; (2) \text{ of } \sin\left(\frac{\pi}{6} + \frac{\pi}{3} \sin \theta\right),$$

as θ changes from 0 to 2π .

2. Prove that

$$(i.) (\sec^2 A + \tan^2 A)(\operatorname{cosec}^2 A + \cot^2 A) = 1 + 2 \sec^2 A \operatorname{cosec}^2 A.$$

$$(ii.) \frac{\sec A + \tan A}{\operatorname{cosec} A + \cot A} - \frac{\sec A - \tan A}{\operatorname{cosec} A - \cot A} = 2(\sec A - \operatorname{cosec} A).$$

3. Having given $\sin A = \frac{4}{5}$ and $\sin B = \frac{8}{17}$, find the value of $\tan(A+B)$.

4. Prove that

$$\begin{aligned} \sin \frac{1}{2}(A-B) \{ \cos A + \cos B - \sin(A+B) \} \\ = \cos \frac{1}{2}(A+B) \{ \sin(A-B) + \cos A - \cos B \}. \end{aligned}$$

5. Find an expression for all the angles which have the same sine as a given angle.

Having given the value of $\sin 2A$, find the number of values of $\tan A$.

6. Prove that

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1} \frac{x+y}{1-xy},$$

and that

$$2 \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{31} = \frac{\pi}{4}.$$

7. Find the sine and cosine of 18° and also the sine and cosine of 54° .

8. If R is the radius of the circumscribing circle of a triangle ABC , prove that

$$\frac{\cos A}{c \sin B} + \frac{\cos B}{a \sin C} + \frac{\cos C}{b \sin A} = \frac{1}{R}.$$

WOOLWICH ENTRANCE EXAMINATION.

9. Write down the expression for the cosine of an angle of a triangle in terms of its sides, and prove that, for a triangle ABC ,

$$\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}.$$

Having given that the sides of a triangle are 35.3, 34.5, and 1.2 find its greatest angle.

10. An observer standing at the distance of 52 feet from the foot of a tower finds that the angle of elevation of the top of the tower is $42^{\circ} 15' 15''$; find the height of the tower, the observer's eye being 5 feet from the ground.

11. A piece of wood is in the form of a regular pyramid on a square base; the side of the base is 6 inches, and the perpendicular distance of the vertex from the base is 8 inches; find the number of cubic inches in the volume of the wood, and the number of square inches in its surface.

12. A cylindrical boiler is hemispherical at its two ends; its radius is 2 feet, and its total length is 8 feet; assuming that a cubic foot of water weighs 62.5 lbs., find the number of tons of water which will fill the boiler. (Take $\pi = 3.14$.)

IV. STATICS AND DYNAMICS.

[The acceleration due to gravity may be taken as equal to 32.]

1. State and prove the proposition known as the "Polygon of Forces."

Forces 1, 2, 3 and $2\sqrt{2}$ act on a point in the directions of the sides AB , BC , CD and the diagonal DB of a square $ABCD$ respectively; determine their resultant, graphically or otherwise.

2. Each of a pair of sculls has four-fifths of its length outside the rowlock, and a man sculling pulls at the handle of each with the force P . Another man thrusts an oar over the stern against the bottom of the water with the force $2P$ at an angle of 60° to the horizon. Compare their effects in propelling the boat.

3. State the conditions of equilibrium of any number of forces acting on a body in one plane.

A smooth wall is inclined at an angle of 60° to the horizon; a heavy uniform rod AB $4\sqrt{6}$ feet long, is in equilibrium at an angle of 45° to the wall, its lower end, A , rests on the wall, and a point in it, C , rests on a smooth horizontal rail parallel to the wall. Draw a diagram showing how the forces act, and find the distance of C from the wall.

4. Find the centre of mass of a uniform triangular board.

The sides AB , BC , CD , DA , of a trapezium are of lengths 54, 36, 27, 45 respectively, AB being parallel to CD . Prove that its centre of gravity is at a distance 16 from AB .

5. Find the ratio of P to W in a smooth screw.

If a power of 1 cwt. acting horizontally and at right angles to the extremity of an arm 8 ft. 4 in. long will raise 5 tons by means of a screw whose axis is vertical and diameter 2 inches, find the inclination to the horizon of the thread of the screw.

6. Find the relation between the power and weight in that system of pulleys in which each string is attached to the weight, the strings being parallel and the weights of the pulleys being neglected.

If there be three pulleys, two of them moveable, and the weight of each $\frac{1}{4}$ lb., what will be the value of P when $W = 8$ lbs.?

7. What is meant by uniform acceleration?

A stone is thrown vertically upwards, with a velocity 36 feet per second. After what times will its velocity be 12 feet a second?

8. A particle moving in a straight line with a uniform acceleration a passes a certain point A with velocity u . State (without proof) the formulae which give (1) the distance s subsequently moved over in time t , and (2) the velocity v with which the particle will be moving when at a distance d from A .

A railway train, moving with velocity 48 miles an hour, has its velocity reduced to 16 miles an hour in 5 minutes. Find the space passed over in the interval, the retardation being assumed to be uniform.

9. Two smooth inclined planes have a common altitude, and their inclinations are 30° and 60° to the horizon. Two masses start simultaneously from the common vertex to fall one down each plane. Compare (1) their times of falling to the bottom, and (2) their final velocities.

10. A particle is projected horizontally from the top of a vertical wall 16 feet high with a velocity $32\sqrt{3}$ feet per second. Find its range on the horizontal ground, and prove that when it strikes the ground its velocity is 64 feet per second.

11. State the law of Newton which tells us how to measure force.

If the unit of force were that which acting on 1 ton of mass would in 1 minute generate a velocity of 1 mile a minute, how many units of force would the weight of 1 ton contain?

A man 10 stone in weight, who pulls with a force equal to his own weight, drags a 10 ton railway carriage from rest on a smooth horizontal line of rails. How far will he move it in one minute?

12. A mass (m), falling vertically, draws a mass M along a smooth horizontal table by means of a fine string which passes over a smooth pulley at the edge of the table; find the tension of the string at any time and the acceleration produced. If the tension is equal to half the weight of m what is the ratio of M to m ?

FURTHER EXAMINATION.

V. MATHEMATICS (1).

[Full marks may be obtained for about four-fifths of this paper. Great importance is attached to accuracy.]

1. Explain the meanings of a , b , u , p , m , c in the following equations representing a straight line :

$$\frac{x}{a} + \frac{y}{b} = 1, \quad x \cos a + y \sin a - p = 0, \quad y = mx + c.$$

What are the relations between these constants, if all three equations represent the *same* straight line?

2. Find the equations of the sides of a triangle, the coordinates of whose vertices are $(1, 4)$, $(2, -3)$, $(-1, -2)$, respectively.

Also find the equations of the bisectors of the angles between the lines

$$12x + 5y - 4 = 0, \quad 3x + 4y + 7 = 0.$$

3. Find the locus of a point P , so that when PM and PV are drawn respectively perpendicular to two fixed lines, OM and ON , the sum of OM and ON is given.

4. Find the locus of a point P (1) when the tangents from P to two given circles are in a given ratio, (2) when the sum of the tangents is constant.

5. Being given the focus and two points on a parabola, find the position of its directrix, and state the number of solutions of the problem.

6. Prove that the normal at any point on an ellipse bisects the angle between the focal vectors of the point.

Draw the normals that pass through a given point on the axis minor of an ellipse, and find when a solution (other than the minor axis itself) is impossible.

7. Show that an ellipse can be projected orthogonally into a circle.

Hence prove that any triangle can be projected orthogonally into an equilateral triangle.

8. In the transformation of the general equation of the second degree from one rectangular system of coordinate axes to another, show that the quantities $a+b$ and $ab-h^2$ remain unaltered, where a , b , $2h$ are the coefficients of x^2 , y^2 , xy respectively.

Find the lengths of the axes of the conic that is represented by the equation

$$12x^2 - 12xy + 7y^2 = 48.$$

9. Find what curve is represented by the equation

$$\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1,$$

and give a figure of the curve.

10. Prove that the sum of the squares of any pair of conjugate diameters of an ellipse is constant.

Being given in magnitude and position two conjugate diameters of an ellipse, find, by geometrical construction, the position and magnitude of its axes.

VI. MATHEMATICS (2).

[Full marks may be obtained for about three-fourths of this paper. Great importance is attached to accuracy.]

1. Solve the equations

$$(i.) (x+4)(x-3) = 5\sqrt{(x+3)(x-2)};$$

$$(ii.) \begin{cases} x^2 - 2xy + 3y^2 = a^2 - 2ab + 3b^2 \\ 3x^2 - 2xy + y^2 = 3a^2 - 2ab + b^2 \end{cases}.$$

2. Find the sum of the series

$$1 + 3 + 6 + \dots + \frac{1}{2}n(n+1).$$

The top and bottom layers of a pile of spherical shot are equilateral triangles, whose sides contain a and b shot respectively. Show that the number of shot in the pile

$$= \frac{1}{6}(b-a+1)\{b^2+b(a+2)+a(a+1)\}.$$

3. Show that

$$\sqrt{x^2+4} - \sqrt{x^2+1} = 1 - \frac{1}{4}x^2 + \frac{7}{64}x^4 - \text{etc.}$$

$$\text{or} = \frac{3}{2x} \left\{ 1 - \frac{5}{4x^2} + \frac{21}{8x^4} - \text{etc.} \right\}$$

4. Show that the expression

$$\frac{1}{b-c} + \frac{1}{c-a} + \frac{1}{a-b}$$

will be positive if a, b, c ; b, c, a , or c, a, b are in descending order of magnitude.

5. Explain how to solve in integers an indeterminate equation of the first degree in two variables.

Find a number of two digits such that if it be multiplied by three, and added to four times the number formed by reversing the digits, the sum is 659.

6. If A and B be two acute angles, such that $\sin A = \frac{4}{5}$ and $\sin B = \frac{5}{13}$, prove (without using tables) that $A+2B$ is greater than 90° and less than 180° .

7. Explain what is meant by Mathematical Induction, and show by means of the identity

$$\cos(n+1)\theta + \cos(n-1)\theta = 2 \cos n\theta \cos \theta,$$

that if $2 \cos \theta = x + \frac{1}{x}$, then $2 \cos n\theta = x^n + \frac{1}{x^n}$, n being a positive integer.

8. A, B are two stations a miles apart, and P a point moving in a straight line. At a given moment the angles BAP, ABP are observed to be α, β , and after a certain interval to be $\alpha + \gamma, \beta - \gamma$. Find the distance described by the point in the interval.

9. Investigate expressions for the radii of the inscribed and circumscribed circles of a triangle in terms of the sides of the triangle.

Find their numerical values in the case of a triangle in which

$$a = 36, \quad B = 73^\circ 15', \quad C = 45^\circ 30'.$$

10. Sum the series

$$\sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots \text{ to } n \text{ terms.}$$

The right angle C of a triangle ABC is divided into n equal parts by lines which meet the hypotenuse AB in points, $P_1, P_2, P_3, \dots P_{n-1}$; show that

$$\frac{1}{CP_1} + \frac{1}{CP_2} + \dots + \frac{1}{CP_{n-1}} = \frac{a+b}{2ab} \left\{ \cot \frac{\pi}{4n} - 1 \right\}.$$

VII. MECHANICS.

[Full marks may be obtained for about three-fourths of the paper. Mathematical instruments must be used for questions requiring a graphical method of solution.]

1. Enunciate and prove the Polygon of Forces.

Five forces OA , OB , OC , OD , OE acting at the point O are in equilibrium. If the forces in OA , OB , OD be respectively 4, 2, 3, find graphically the forces in OC and OE . Given

$$\angle AOB = 45^\circ, \angle AOC = 120^\circ, \angle AOD = 210^\circ, \angle AOE = 270^\circ.$$

2. A rod ACB weighing 25 ozs. rests upon a smooth peg C , and its end A is attached to a fixed point O in the same horizontal line with C by means of a string OA . Find graphically the position of the centre of gravity of the rod and the magnitudes of the tension of the string and the pressure between the rod and peg.

$$AC = 4\frac{1}{2}, \quad AO = 2\frac{1}{2}, \quad OC = 2\frac{1}{2} \text{ inches.}$$

3. A , B , C are three fixed smooth pegs in a vertical plane, A being 3 feet vertically below B and 4 feet horizontally to the right of C . A string 13 feet long passes round the three pegs and has its extremities attached to a weight W ; find the tension of the string and the resultant pressure on each peg.

4. The weights of a system of heavy bodies are $W_1, W_2, \dots$ and the coordinates of their centres of gravity are $x_1, y_1; x_2, y_2; \dots$ respectively. Find the coordinates of the centre of gravity of the system.

A square board is divided into 36 equal squares by lines drawn parallel to the sides. A figure is formed by taking the 1st and 4th squares in the first column, the 1st, 2nd, and 4th squares in the second column, the 2nd, 3rd,

4th, and 5th squares in the third column, the 4th, 5th, and 6th squares in the fourth column, the 2nd, 3rd, 4th, 5th, and 6th squares in the fifth column, and the 3rd, 5th, and 6th squares in the last column. Show that the centre of gravity of the figure formed of these 20 squares divides the line AB in the ratio of 11 to 13.

5. A uniform rod rests on the rim of a plate with its middle point at the centre of the rim. The plate itself rests on a horizontal plane. A downward vertical force, just sufficient to disturb equilibrium, being applied to one end of the rod, show that the plate and rod will begin to move together, or the rod only, according as the weight of the rod is greater or less than $\frac{2}{3}$ of the weight of the plate; $4a$, $5a$ being the diameters of the base and rim of the plate respectively, and $6a$ the length of the rod.

6. Define *work*, and show how it is measured.

A right-angled triangle ABC turns stiffly in its own plane about the middle point of the hypotenuse AB . If forces P , Q , R , just sufficient to overcome the resistance, be applied at right angles to the sides BC , CA , AB at the angular points, all tending to turn the triangle in the same direction, show that the work done by them in turning the triangle through a right angle $= \frac{1}{2}\{Rc + (P - Q)(a - b)\}$; the forces remaining throughout the motion parallel to their original directions and constant in magnitude.

7. A heavy particle slides down a rough inclined plane; find the space described from rest in a given time.

Two particles are projected with a velocity of 40 feet per second from points 88 feet apart, the one up and the other down, a rough plane ($\mu = \frac{1}{4}$) inclined to the horizon at an angle $\tan^{-1} \frac{3}{4}$. Find when and where they will meet, and account for the double solution [$g = 32$].

8. Within a smooth circular tube fixed in a vertical plane are two particles of mass P , Q connected by a string whose length is equal to half that of the tube. Find the acceleration of each particle in the direction of motion, and the tension of the string supposed tight, when the line joining the particles makes an angle θ with the horizon.

9. Two smooth imperfectly elastic balls, moving in one plane with given velocities in given directions, impinge obliquely on each other; determine the motion of each after impact.

10. A particle hangs from a fixed point in a wall by a string of length a , find the least velocity which must be given to it in order that it may make a complete revolution, without the string becoming slack.

If the string come in contact with a nail in the wall situated in the horizontal line through the point of suspension and at a distance b from it, find the least initial velocity in order that the particle may make a complete revolution round the nail, without the string becoming slack.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1892.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.]

1. The sum of any two sides of a triangle is greater than the third side, and their difference is less than the third side.

2. If two quadrilaterals $ABCD$, $EFGH$ have the four angles A , B , C , D respectively equal to the four angles E , F , G , H ; and likewise the sides AB , DC respectively equal to the sides EF , HG ; show that if AD and BC meet when produced the quadrilaterals are equal in every respect.

3. Define *parallel straight lines*; and show that parallelograms on the same base and between the same parallels are equal to one another.

WOOLWICH ENTRANCE EXAMINATION.

4. Enunciate the proposition which is represented in algebraical symbols by

$$(a+b)^2 + a^2 = 2a(a+b) + b^2;$$

and give the construction by which the proposition is proved.

If in a quadrilateral $ABCD$, the sides AD and BC are each perpendicular to the side AB , show that the square on DC is less than the sum of the squares on the three other sides by twice the rectangle contained by AD and BC .

5. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

6. Define *circle*, *tangent*, *angle in a segment*.

If a point be taken within a circle, from which there fall more than two equal straight lines to the circumference, that point is the centre of the circle.

7. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle.

If the bisectors of the adjacent angles A and B of a quadrilateral $ABCD$ meet in E , and the bisectors of the adjacent angles B and C , C and D , D and A meet respectively in F , G , H ; show that a circle can be described round the quadrilateral $EFGH$.

8. If from any point without a circle there be drawn two straight lines, one of which cuts the circle, and the other meets it, and if the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, be equal to the square on the line which meets the circle, the line which meets the circle shall touch it.

9. About a given circle, describe a triangle equiangular to a given triangle.

10. Inscribe an equilateral and equiangular hexagon in a given circle.

Show that if $A_1A_2A_3A_4 \dots$ be an equilateral and equiangular figure of 24 sides, the straight lines A_1A_{10} and A_5A_{18} are at right angles.

11. If two triangles have one angle of the one equal to one angle of the other, and the sides about the equal angles proportionals, the triangles shall be equiangular to one another, and shall have those angles equal which are opposite to the homologous sides.

12. Similar triangles are to one another in the duplicate ratio of their homologous sides.

If C be the centre of a circle, $OPCQ$ a straight line cutting the circle in P and Q , OT a tangent to the circle, and PN another tangent cutting OT in N ; show that the triangles OPN and OTC bear the same ratio to one another that OP bears to OQ .

II. ALGEBRA.

(Up to and including the Binomial Theorem ; the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Multiply

$$a^3 + 2a^2b + 3ab^2 + 3b^3 \quad \text{by} \quad a^3 - 3a^2b + 3ab^2.$$

Also show that

$$(x-4y)^3 - (y-4x)^3 + (2y-3x)^3 - (2x-3y)^3 = 30(x-y)(x+y)^2.$$

2. Divide

$$x^6 - (p^4 - 2pq)x^4 - p^2q^2x^2 - q^4 \quad \text{by} \quad x^3 - p^2x^2 + pqx - q^2.$$

3. Find the highest common factor of

$$4x^4 - 29x^2 + 25 \quad \text{and} \quad 2x^4 + x^3 - 14x^2 - 4x + 15.$$

Also resolve each of these expressions into the simplest possible factors.

4. Simplify

$$(i.) \quad \frac{3}{1-x} - \frac{1}{2-x} - \frac{1-2x}{1-x+x^2};$$

$$(ii.) \quad \frac{a^2}{\sqrt{a^2+b^2}-b} - \frac{b^2}{\sqrt{a^2+b^2}+a}.$$

Also find the value of

$$\frac{1}{x-2a} + \frac{2}{x+b} + \frac{1}{b}, \quad \text{when } x = \frac{3ab}{b-a}.$$

5. A square of carpet is cut up into strips so as to cover a border $(a+b)$ feet wide round the floor of a room, whose length is $10a$ feet and breadth $10b$ feet. What was the length of the side of the square?

6. Solve the equations

$$(i.) \quad \frac{2x-3y}{4} = \frac{3x-2y}{5} = 7\frac{1}{2};$$

$$(ii.) \quad \frac{\sqrt{3x-2} - \sqrt{x+7}}{\sqrt{3x-2} + \sqrt{x+7}} = \frac{x-3}{2x+1}.$$

7. Find the sum, product, and difference of the roots of the quadratic equation

$$x^2 - 2bx + c^2 = 0.$$

Write down an equation whose roots are each equal to the sum of the reciprocals of the roots of the given equation.

8. A cask was filled with wine and water mixed together in the proportion of 5 : 3. When 16 gallons of the mixture had been drawn off and the cask filled up with water, the proportion became 3 : 5. How many gallons did the cask hold?

9. The first and last of 46 terms in Arithmetical Progression were -5 and $+25$ respectively. Find the two middle terms and the sum of all the terms.

Also find two numbers whose arithmetic mean exceeds their geometric mean by 2, and whose harmonic mean is one-fifth of the larger number.

10. A purse contains a sovereign, a half-sovereign, a half-crown, a florin, a shilling, a threepenny piece, and a penny. How many different sums of money can be obtained by taking out four of the coins?

If two sovereigns were added to the contents of the purse, how many different sums could then be obtained by taking out four coins?

11. Write down the continued product of the n factors

$$1 + x_1, \quad 1 + x_2, \quad 1 + x_3, \quad 1 + x_4, \quad \dots \quad 1 + x_n,$$

and deduce the expansion of $(1+x)^n$ by the Binomial Theorem.

Also write down the twelfth term and the middle term in the expansion

of
$$\left(a - \frac{1}{a}\right)^{16}.$$

12. Explain how the different powers and roots of numbers may be found by means of logarithms.

What is the smallest number of terms which may be taken of the Geometric Progression 4, 6, 9, ... to have a sum exceeding 8000?

III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Compare the magnitudes of the unit of circular measure and the degree.

Find correct to one second, the time between one and half-past one o'clock when the circular measure of the angle between the hands is $\frac{1}{2}$.

2. Express all the trigonometrical functions of an angle in terms of the tangent.

If $\tan \theta = \frac{x \sin \phi}{1 - x \cos \phi}, \quad \tan \phi = \frac{y \sin \theta}{1 - y \cos \theta},$

prove that $\frac{\sin \theta}{\sin \phi} = \frac{x}{y}.$

3. Find $\sin(A+B)$ in terms of sines and cosines of A and B . Find also $\sin 3A$ and $\cos 2A$ in terms of $\sin A$, and if A is less than 180° , prove that $\sin 4A$ is less than $4 \sin A$.

4. Find the smallest positive values of A and B from the equations

$$\begin{cases} \tan A + \tan B = \cot A + \cot B, \\ A - B = 60^\circ. \end{cases}$$

5. Prove that

$$\sin A - \sin B = 2 \sin \frac{A-B}{2} \cos \frac{A+B}{2}.$$

Prove also that

$$\sin 5A - \sin 3A = 2 \sin A - 4 \sin A \sin^2 2A.$$

6. Find $\tan(A+B+C)$ in terms of the tangents of A , B , and C .

Prove that in any triangle ABC

$$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1.$$

7. In every triangle the sides are proportional to the sines of the opposite angles.

If the cosines of two of the angles are inversely proportional to the opposite sides, prove that the triangle is either right-angled or isosceles.

8. Explain the notation $\sin^{-1}a$.

If $\tan^{-1}a = \operatorname{cosec}^{-1}a = \cos^{-1}b$, prove that one value of b is

$$\frac{\sqrt{5}-1}{2}.$$

9. Find the area, and radius of the inscribed circle, of a triangle whose sides are 706, 690, and 240 feet.

10. From two stations C and D , on the same side of the line AB , joining the two objects A and B , whose distance apart is known, the angles ACB , BCD , CDB , and ADB are observed, show how CD may be found.

If $ACB = 90^\circ$, $ADB = 75^\circ$, $BCD = 30^\circ$, $CDB = 105^\circ$, prove that

$$CD = \frac{2AB}{\sqrt{8+2\sqrt{3}}}.$$

11. The height of a conical tent is $7\frac{1}{2}$ feet, and it is to inclose 200 square yards of ground, find how much canvas will be required. ($\pi = \frac{22}{7}$.)

12. The silk covering of an umbrella forms a portion of a sphere of $3\frac{1}{2}$ feet radius, the area of the silk being $14\frac{2}{3}$ square feet. Find the area of the ground sheltered from vertical rain when the stick is held upright. ($\pi = \frac{22}{7}$.)

IV. STATICS AND DYNAMICS.

[The acceleration due to gravity may be taken as equal to 32 foot-second units.]

1. If the side of a regular hexagon $ABCDEF$ represents a force of 100 lbs., find, to the nearest pound, the magnitudes of the forces represented by the straight lines AE , AD , FB ; and, supposing them to act at a point, show how to draw, by the Polygon of Forces, the line representing their resultant.

2. A uniform rod AB which can turn freely in a vertical plane about a hinge at A , is kept in a horizontal position by a string BC attached to a fixed point C in the vertical plane, the angle ABC being obtuse. Show in a diagram the forces acting on the rod, and prove that two of them are equal.

3. State the conditions of equilibrium of a system of forces acting in one plane on a particle (i.) when the particle is otherwise free, and (ii.) when it is constrained to move on a smooth straight rod in the plane of the forces.

A ring C rests upon a fixed smooth horizontal rod AB , whose length is 13 feet. To the ring are attached two strings, one of which is 7 feet long and has its other extremity fixed at A , the other passes over a smooth hook 8 feet below B and supports a weight of 20 lbs. Find the tension of the string AC .

4. Define the *centre of gravity* of a body; and show how to find the position of the centre of gravity of a body formed of two parts whose weights and centres of gravity are given.

A line BOC is bisected at right angles by a line AOD , the lengths of AO , OD being 5 and 7 inches respectively; find the distance of the centre of gravity of the figure $ABDC$ from O .

5. Point out the use of a single smooth pulley (i.) when fixed, (ii.) when moveable, and in the latter case investigate the relation between the power and the weight when the angle between the portions of the string diverging from the pulley is α . Within what limits must α lie in order that there may be mechanical advantage?

6. Two weightless levers AOB , COD , whose lengths are 8 and 9 inches respectively, are jointed together at O four inches from B and D . If A , C be connected by a string 3 inches long, and B , D be pulled apart by forces P , P , in the straight line BD , find the tension of the string.

Draw a diagram showing the forces acting on the lever AOB .

7. Enunciate the First and Second Laws of Motion, and give examples of force producing change of motion as regards (i.) magnitude only, (ii.) direction only, and (iii.) both magnitude and direction.

8. Compare the velocities of two particles, one of which describes $4\frac{1}{2}$ miles in 35 minutes, and the other 77 feet in 2 seconds.

9. A particle moves from rest under the action of a uniform force; express the velocity acquired, the acceleration, and the force, in terms of s the space described, t the time, and m the mass of the particle.

A smooth tube ACB , consisting of two straight portions AC , CB , with a bend at its lowest point C , is fixed in a vertical plane, and a particle acted on by gravity starts from rest from a point P in the arm AC , and, passing the bend C without change of velocity, rises in the arm CB to a point Q . Show that PQ is horizontal, and that the spaces CP , CQ are proportional to the times in which they are described.

10. If a force equal to the weight of 21 lbs. act upon a body, and generate in it in one minute a velocity of 3 feet per second, find the mass of the body.

11. A particle is projected with a velocity of 80 feet per second in a direction making an angle of 30° with the horizon; find the greatest height attained, and the range on the horizontal plane through the point of projection.

Find also the velocity of the particle and its distance from the point of projection at the end of two seconds.

FURTHER EXAMINATION.

V. PURE MATHEMATICS.

[Full marks may be obtained for about three-fifths of this paper. No Candidate must attempt more than ten questions. Great importance will be attached to accuracy.]

1. Prove that the sum of the squares of any three real quantities, which are not all equal, is greater than the sum of the products of every two of the quantities.

2. Expand

$$\frac{(1-x)^2}{1+x+x^2}$$

in a series of ascending powers of x .

If a_r be the coefficient of x^r in this expansion, prove that

$$a_{3n} = 0, \quad a_{3n+2} - a_{3n+1} = 6.$$

3. Find two numbers whose product exceeds the square of the smaller by the double of the greater. (Only one solution is required.)

4. In a certain triangle ABC the base AB and the perpendicular altitude are each equal to the radius of the circumscribing circle. Prove that

$$ab = 2c^2 \quad \text{and} \quad \cot A + \cot B = 1.$$

5. A vertical column stands on an inclined plane. The elevation of the top of the column above the horizon is found to be 45° at points on this plane respectively a ft. and b ft. from the foot of the column along the line of greatest slope through the foot. Show that the inclination of the plane to the horizon is

$$\tan^{-1}\left(\frac{a-b}{a+b}\right),$$

and that the square of the height of the column is the harmonic mean between a^2 and b^2 .

6. Show that θ , the circular measure of an angle not exceeding half a right angle, is equal to the sum of either of the infinite series

$$\tan \theta - \frac{1}{3} \tan^3 \theta + \frac{1}{5} \tan^5 \theta - \frac{1}{7} \tan^7 \theta + \dots,$$

$$\sin 2\theta - \frac{1}{2} \sin 4\theta + \frac{1}{3} \sin 6\theta - \frac{1}{4} \sin 8\theta + \dots$$

7. The tangent at any point P of a conic section, whose focus is S , meets the directrix in D . Show that the angle PSD is a right angle.

If in a parabola the abscissa of P be one-twelfth of the latus rectum, PD will be bisected by the axis.

8. If $SY, S'Y'$ be the perpendiculars from the foci of an ellipse on the tangent at a point P , show that $SY \cdot S'Y' = BC^2$.

Show also that, if the circle on YY' as diameter cut the normal at P in R , and PN be the ordinate at P , $PN:PR::BC:SC$.

9. Prove that the difference of the squares of any two conjugate diameters of a hyperbola is constant.

If the base and one of the equal sides of an isosceles triangle be the asymptotes of a hyperbola which touches the other side, prove that the difference of the squares of any two conjugate diameters is double of the square of the base of the triangle.

10. Express the area of a triangle in terms of the rectangular co-ordinates of its angular points.

Hence find the area of the triangle formed by joining the points, whose coordinates are $(a, c+a)$, (a, c) , $(-a, c-a)$, and verify your result geometrically.

11. AOA', BOB' are two perpendicular diameters of a circle, and M, N are the middle points of OA, OB respectively. Taking these diameters as axes of coordinates, write down (1) the equations of the straight lines $AN, B'M$; (2) the equations of the radii drawn to the two points where $AN, B'M$ produced cut the circumference of the circle.

What is the angle between the two radii?

12. Prove that the polar equation of a parabola, with the focus as pole, may be written in the form

$$r \cos^2 \frac{\theta}{2} = a.$$

Prove also that the equation of the polar of a point, whose coordinates are (r', θ') , is

$$rr' \sin \theta \sin \theta' + 2a(r \cos \theta + r' \cos \theta') = 4a^2.$$

13. Find the equation of the straight line joining two points on an ellipse whose eccentric angles are given.

If tangents are drawn to the ellipse at two points, whose eccentric angles differ by 120° , show that their point of intersection will always lie on a similar and similarly situated ellipse, whose axes are twice as long as those of the given ellipse.

14. Find the equation of a hyperbola, of eccentricity $\frac{5}{4}$, which has one focus at the point $(a, 0)$, and the equation of the corresponding directrix $4x - 3y = a$.

Find also the coordinates of the centre, and the equation of the other directrix.

VI. MECHANICS.

[Full marks may be obtained for about two-thirds of this paper. Great importance is attached to accuracy. N.B.— g may be taken = 32.]

1. If OA and OB represent two forces in direction and magnitude, find the line OR representing their resultant.

If OC and OD be two equal lines cut off from OA and OB respectively, and if OR meet CD in G , find the ratio of CG to GD .

2. A rod AB , whose centre of gravity is at G , is supported at an angle of 42° to the vertical, with one end A in contact with the smooth vertical wall AD , by a string attached to the point C of the rod and also to a point in the wall.

$$AB = 3'', \quad AC = 1'', \quad AG = 1\frac{5}{8}''.$$

Find graphically the length of the string and its tension in terms of the weight of the rod, which is 55 ozs.

3. The horizontal rod BEC and the rod DE being without weight, and the latter perfectly rigid, find graphically the actions at B and E when the weight 150 lbs. is suspended from C . BD is vertical and

$$BC = 1.8, \quad BE = .9, \quad DE = 1.5, \quad BD = 1.2.$$

4. Find the relation between the power and the weight in the smooth wedge whose transverse section is an isosceles triangle. If the pressure between the wedge and the separated points in contact with it be proportional to the distance between these points, prove that the work done is proportional to the square of the distance of penetration.

5. The uniform square $ABCD$ rests vertically with the side BC upon a horizontal plane, coefficient of friction $\frac{1}{2}$, and has a rope attached at D and passing over a small smooth pulley at the point E in BA produced till EA is equal to AB . If the rope be pulled, find whether the initial motion of the square will be tilting or sliding.

6. Apply the laws of motion to determine the path of a projectile.

From a given point in a railway carriage, moving with uniform velocity in a straight line, bullets are fired continuously with a constant velocity at right angles to the rails, and with a constant inclination to the horizon. Find the locus at any instant of all the bullets which have not reached the ground.

7. Find the acceleration down a smooth inclined straight line.

Two particles slide down two straight lines, in the same vertical plane, at right angles to one another, starting simultaneously from their point of intersection; prove that their distance apart, at any time, will be equal to the distance either would have descended vertically in that time.

8. What is the experimental principle assumed in determining the velocities of two given elastic balls after direct impact? If the elasticity is perfect, prove that the total kinetic energy is unaltered by impact.

Prove also the converse of this proposition.

9. Two equal and perfectly elastic spherical beads, each of radius r , strung upon an inextensible string, are placed on a smooth table and are drawn apart to the greatest possible distance a between their centres. One of them is then projected directly towards the other with a given velocity u . Determine their distance apart, and their velocities, at any time t from projection.

Write down the equation to the path of their centre of gravity, supposing the table removed at the instant of projection.

10. Two given weights are connected by a string passing over a smooth pulley. Determine their acceleration and prove that the resultant pressure between the string and pulley is less than it would have been if half the sum of the weights had been suspended at each end of the string.

11. Define the terms Work, and Energy (potential and kinetic), stating the units commonly used in each case.

A shot of 6 lbs. weight leaves the muzzle of a gun of 6 cwt. in a horizontal direction with the velocity of 1000 feet per second. Find the potential energy of the charge in the ordinary units, assuming that the gun is perfectly rigid, its bore smooth, and the carriage free to move on a horizontal plane.

How would the conclusion be affected in the absence of any one of these assumptions.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1892.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[*Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Make a triangle of which the sides shall be equal to three given straight lines ; and show that if ABC be a triangle with the vertical angle at A greater than either of the base angles, it is not possible to form a second triangle with sides equal to those of ABC and base equal to twice BC .

2. Give Euclid's *Axiom on Parallels*, and those on the addition and subtraction of equals to and from equals and unequals.

Prove that if a straight line fall on two parallel straight lines, it makes the alternate angles equal to one another ; and the exterior angle equal to the interior and opposite angle on the same side ; and also the two interior angles on the same side together equal to two right angles.

3. ABC being an isosceles triangle, and D any point in the base BC ; show that the perpendiculars to BC through the middle points of BD and DC divide AB and AC at H and K respectively, so that $BH=AK$ and $AH=CK$.

WOOLWICH ENTRANCE EXAMINATION

4. If a straight line be bisected, and produced to any point, the rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

Prove this ; and give its equivalent in algebraical symbols.

5. Prove that in every triangle, the square on the side subtending an acute angle, is less than the squares on the sides containing that angle, by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle, and the acute angle.

If in a quadrilateral $ABCD$, the square on AB is greater than the squares on the three other sides BC , CD , DA by twice the rectangle CD , DA , show that the angle ACB is obtuse.

6. Prove that the straight line drawn at right angles to the diameter of a circle, from the extremity of it, falls without the circle ; and that no straight line can be drawn from the extremity, between that straight line and the circumference, so as not to cut the circle.

Give the corollary to this proposition.

7. Prove that the angle at the centre of a circle is double of the angle at the circumference on the same base, that is, on the same arc.

If the diagonals of a quadrilateral inscribed in a circle cut at right angles, show that the angles which a pair of opposite sides of the quadrilateral subtend at the centre of the circle are supplementary.

8. If from any point without a circle two straight lines be drawn, one of which cuts the circle, and the other touches it ; prove that the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, is equal to the square on the line which touches it.

9. Inscribe a circle in a given triangle ; and indicate the construction when the circle is required to touch one side externally.

10. Inscribe a regular pentagon in a given circle.

If a regular pentagon and a regular hexagon be on the same base and on the same side of it, prove that the pentagon is wholly within the hexagon.

11. In a right-angled triangle, if a perpendicular be drawn from the right angle to the base, prove that the triangles on each side of it are similar to the whole triangle, and to one another.

If AB, BC be two sides of a regular figure, L and M their respective middle points, and O the centre of the inscribed circle, show that the triangle BLM has to the triangle OLM the duplicate ratio of that which a side of the figure has to the diameter of the circle.

12. Prove that in equal circles, angles, whether at the centres or circumferences, have the same ratio which the circumferences on which they stand have to one another.

II. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Multiply

$$4x^2 + 2(3a - 2b)x + 9a^2 \quad \text{by} \quad 2x - 3a,$$

and divide

$$16a^4 - 8a^2bc + b^2c^2 \quad \text{by} \quad 2a - \sqrt{bc}.$$

2. Extract the square root of

$$9a^6 - 12a^5b + \frac{10a^3b^3}{3} - \frac{4ab^5}{27} + \frac{b^6}{81}.$$

3. Define the meaning of the symbol $\frac{a}{b}$, and from your definition

show that $\frac{a}{b} = \frac{na}{nb}$, where a, b, n are positive integers.

Arrange in order of magnitude, a, b, n being positive integers,

$$\frac{na}{(n+1)b}, \quad \frac{(n+1)a}{nb}, \quad \frac{na}{(n-1)b}, \quad \frac{(n-1)a}{nb},$$

and justify your arrangement.

4. Simplify

$$(i.) \quad \sqrt[m]{a^{m^2} \div (a^m)^2},$$

$$(ii.) \quad \left\{ \frac{1}{2x^2 - 3ax - 2a^2} + \frac{1}{2x^2 - 5ax + 2a^2} \right\} \\ \div \left\{ \frac{1}{2x^2 + 3ax - 2a^2} - \frac{1}{2x^2 + 5ax + 2a^2} \right\},$$

$$(iii.) \quad \frac{a^3 - b^3}{a - b + \frac{2(a+b)}{\frac{a+b}{b} - \frac{a}{a-b}}}.$$

5. Find the value of

$$\left(\frac{x+a}{x-a} \right)^2 - \frac{x}{3a} \quad \text{when} \quad x = a(1 + 2\sqrt{3}),$$

and reduce

$$\sqrt{6 - \sqrt{17 - 12\sqrt{2}}}$$

to the simplest possible surd form.

6. Solve the equations

$$(i.) \quad \frac{x}{x+2} - \frac{3x-10}{3x-9} = \frac{5}{3-x},$$

$$(ii.) \quad 2ax - by = a^2, \quad bxy = a^3,$$

$$(iii.) \quad \sqrt{5x-14} + \sqrt{3x-13} = 2\sqrt{x} + \sqrt{2x+1}.$$

7. Prove that a quadratic equation cannot be satisfied by more than two values of the unknown quantity.

In a certain quadratic the coefficients of x^2 and x are 1 and 2 respectively, and the addition of 8 to each of the roots changes the sign but not the magnitude of the third term. Find the original quadratic, and the coefficient of x in the transformed equation.

8. Insert five arithmetic means between $a - 2b$ and $3a + b$.

The last term of an arithmetic progression is ten times the first, and the last but one is equal to the sum of the 4th and 5th. Find the number of the terms, and show that the common difference is equal to the first term.

9. Find the number of arrangements that can be made of the letters of the word *infinite*, (1) when they are taken all together, (2) when they are taken four together so that each arrangement has two vowels and two consonants.

10. Write down the first five terms in the expansion of $(1-4x)^{\frac{1}{2}}$ by the Binomial Theorem, and show that the coefficient of x^n may be written in the form

$$-\frac{2 \cdot \overline{2n-2}}{\overline{n} \cdot \overline{n-1}}.$$

11. Prove that

$$\log 20 + 7 \log \left(\frac{15}{16} \right) + 5 \log \left(\frac{24}{25} \right) + 3 \log \left(\frac{80}{81} \right) = 1,$$

and, by means of the logarithm tables supplied, find the fifth root of

$$\frac{66901 \times 337}{7824}$$

correct to five places of decimals.

III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy.]

1. Given that $\sin A = \frac{3}{5}$; find the other trigonometrical ratios of A .

2. Trace the changes in magnitude and sign of

$$(i.) \cot \theta, \text{ and } (ii.) \cos \theta - \sqrt{3} \sin \theta,$$

as θ varies from 0 to 2π .

3. Prove that

$$(i.) \sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \cdot \operatorname{cosec}^2 A ;$$

$$(ii.) \frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2 ;$$

$$(iii.) \tan\left(\frac{\pi}{4} + A\right) - \tan\left(\frac{\pi}{4} - A\right) = 2 \tan 2A.$$

4. Obtain a formula giving the value of $\sin \frac{A}{2}$ in terms of $\sin A$.

Show how to get rid of the ambiguities of sign when A lies between 270° and 450° .

5. Prove that

$$(i.) \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2},$$

$$(ii.) \cos B - \cos A = 2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}.$$

Prove also that

$$(i.) \sin A \sin(B-C) + \sin B \sin(C-A) + \sin C \sin(A-B) = 0,$$

$$(ii.) \sin(B-C) \sin(B+C-2A) + \sin(C-A) \sin(C+A-2B) + \sin(A-B) \sin(A+B-2C) = 0.$$

6. Find an expression for $\sin 72^\circ$, and calculate its value correctly to three places of decimals.

7. Prove that

$$\tan^{-1} \frac{2mn}{m^2 - n^2} + \tan^{-1} \frac{2pq}{p^2 - q^2} = \tan^{-1} \frac{2MN}{M^2 - N^2},$$

where

$$M = mp - nq, \quad N = np + mq.$$

8. Explain how to solve a triangle, having given two sides and the included angle.

Given that $b = 23$, $c = 32$, $A = 63^\circ$; find a as nearly as you can, by aid of the logarithm tables supplied.

9. Find an expression for $\tan \frac{A}{2}$ in terms of the sides of the triangle ABC .

Prove that, if Δ is the area of the triangle ABC ,

$$\frac{\tan \frac{A}{2}}{(a-b)(a-c)} + \frac{\tan \frac{B}{2}}{(b-c)(b-a)} + \frac{\tan \frac{C}{2}}{(c-a)(c-b)} = \frac{1}{\Delta}.$$

10. The angle of elevation of the top of a tower at a certain spot is 55° , and at a spot, in the same horizontal plane, 25 yards further from the tower the angle of elevation is 47° . Find the height of the tower.

11. Find the area of the surface (including the ends) of a hexagonal prism, whose height is 8 ft., the base being a regular hexagon with a side of length 3 ft.

12. The radii of the internal and external surfaces of a hollow spherical shell of metal are 3 ft. and 5 ft. respectively. If it be melted down and the material formed into a cube, find an approximate value for the length of an edge of the cube.

IV. STATICS AND DYNAMICS.

[The acceleration due to gravity may be taken as equal to 32 foot-second units.]

1. Explain how the resultant of two known forces acting on a particle may be determined by a geometrical construction.

If D be the middle point of the base BC of a triangle ABC , and the resultant of the forces represented by BA , BD be equal to the resultant of those represented by CA , CD , show that the triangle ABC is isosceles.

2. A , B , C are three points in a straight line ABC . Draw a diagram representing the directions of three parallel forces P , Q , R in equilibrium acting at these points respectively. State also the necessary and sufficient conditions of equilibrium.

A uniform rod, 2 feet long and weighing 3 lbs., lies on a horizontal plane; find the least force which, applied 5 inches from one end, will raise that end above the plane.

3. Define the moment of a force about a point, and show how it can be geometrically represented.

The side BC of a triangle ABC is bisected in E and produced to F . Show that the sum of the moments about F of the forces represented by AB , AC is equal to twice the moment about F of the force represented by AE .

4. Show that the centre of gravity of a triangular lamina coincides with that of three equal weights placed at its angular points.

A lamina in the form of a right-angled triangle ABC is suspended from the right angle C . If $CA = 2$ feet, and $CB = 3$ feet, find the weight which must be suspended from A in order that AB may be horizontal in the position of equilibrium, the weight of the lamina being 12 lbs.

5. A smooth rod BC is passed through a small ring and placed upon a horizontal plane, with its ends attached to a fixed point A in the plane by two strings AB , AC , which are tight. A horizontal force being applied to the ring, find its direction, and also the position of the ring on the rod, in order that equilibrium may not be disturbed, the lengths of BC , CA , AB being 25, 20, and 15 inches respectively.

6. Define a machine, and point out its use as regards work done by it.

Find the greatest vertical height through which a force of 150 lbs. can raise a weight of 4 cwt. by drawing it up a smooth, sloping plank 20 feet in length.

7. Prove that, if an insect crawl along the minute hand of a clock with a velocity equal to that of the extremity of the hand, it will pass from one end to the other in 9 minutes 33 seconds nearly.

8. Explain, with examples, what is meant by a particle having two or more velocities in different directions at the same instant.

A particle has two velocities, $3u$ in a direction from A to B , and $8u$ in a direction from C to A , ABC being an equilateral triangle. Find the magnitude of its resultant velocity.

9. State the expression for the space described in time t by a particle projected with velocity u , and subject to an acceleration a in the direction of projection. If v be the final velocity, find the space in terms of v , t , and a .

If $u = 20$ and $a = 12$, the units of time and space being one second and one foot, find the space described in the 4th second.

10. What is meant by the statement that g , the acceleration due to gravity, is equal to 32 nearly?

Show how you would roughly obtain the value of g by observing the motion of two weights P , Q , connected by a string passing round a fixed smooth pulley.

Describe fully a single observation which would give the value of g ; P , Q being 47 and 49 ounces respectively.

11. A particle is projected in any direction not vertical; explain clearly why it does not proceed to describe a straight line.

The engine driver of an express train throws a ball vertically upwards; show in a diagram the actual path of the ball.

FURTHER EXAMINATION.

V. PURE MATHEMATICS.

[Full marks may be obtained for about three-fifths of this paper. No Candidate must attempt more than ten questions. Great importance will be attached to accuracy.]

1. A certain council consists of a chairman, two vice-chairmen, and twelve other members. How many different committees of six members can be chosen, including always the chairman and *at least* one of the vice-chairmen?

Show that 8190 is the total number of different committees which can be formed consisting of the chairman, one (and only one) of the vice-chairmen, and at least one of the other twelve members.

2. Express as a continued fraction the quotient obtained by dividing

$$\frac{1}{a^2 - \frac{1}{2a^2 - \frac{1}{2a^2 - \frac{1}{2a^2 - \dots}}}} \dots \text{ by } \frac{1}{a - \frac{1}{2a - \frac{1}{2a - \frac{1}{2a - \dots}}}}.$$

3. There are six balls in a bag, each of which is known to be either white or black. The first five balls drawn are three white and two black. What is the chance that the other ball is white?

If the five balls were replaced, and a second drawing gave also three white and two black. What would the chance then be?

4. A hill in the shape of a right cone stands on a horizontal plane. At a certain point in the plane the circular base of the cone subtends a right angle, and the elevation of the summit is half a right angle. Show that the slant side of the hill, as seen against the sky, subtends 60° at the same point.

5. The lengths of the sides of a right-angled triangle are in arithmetical progression. Find the ratios of the sides, and show that the diameters of the escribed circles are in harmonical progression.

6. Assuming the truth of De Moivre's theorem, prove that

$$\sin \theta = \theta - \frac{\theta^3}{3} + \frac{\theta^5}{5} - \text{etc.}$$

Also prove that

$$\theta = x \sin \theta + \frac{x^2 \sin 2\theta}{2} + \frac{x^3 \sin 3\theta}{3} + \text{etc.},$$

where $2x = \sec \theta$.

Are these formulæ true for all values of θ ?

7. Show how to describe a conic section, whose focus, directrix, and eccentricity are given.

If a series of conics pass through two fixed points, P, Q , and if a directrix of each pass through a fixed point in PQ produced, prove that the corresponding foci of the conics all lie on a fixed circle.

8. If the ordinate, tangent, and normal at any point of an ellipse meet the major axis in N, T, G , respectively, prove that

$$(i.) \quad NG : CN :: BC^2 : AC^2.$$

$$(ii.) \quad SC \text{ is a mean proportional between } CG \text{ and } CT.$$

9. Prove that the latus rectum of an hyperbola is a third proportional to the transverse and conjugate axes.

If SY , the perpendicular from the focus upon an asymptote, be produced to meet the conjugate axis in W , prove that YW is a third proportional to the conjugate and transverse semi-axes.

10. Show how to express the position of a point on a plane by means of polar coordinates.

Indicate in a figure the four points whose polar coordinates are

$$\left(a, \frac{\pi}{3}\right), \quad \left(2a, -\frac{\pi}{2}\right), \quad \left(-a, \frac{\pi}{6}\right), \quad \text{and} \quad \left(-2a, -\frac{2\pi}{3}\right),$$

respectively, and also the locus of the equation $r(4 - 3 \sin^2 \theta) = 8a \cos \theta$.

11. Find the equations of the two straight lines drawn through the point $(0, a)$, on which the perpendiculars let fall from the point $(2a, 2a)$ are each of length a .

Show that the equation of the straight line joining the feet of these perpendiculars is

$$y + 2x = 5a.$$

12. Define the terms *pole* and *polar* with respect to a conic section, and find the equation of the polar of any point with respect to a parabola.

If a perpendicular be let fall from the pole upon the polar, prove that the distance of the foot of this perpendicular from the focus is equal to the distance of the pole from the directrix.

13. Find the equation of the tangent at any point of the ellipse

$$a^2y^2 + b^2x^2 = a^2b^2.$$

If the tangent at P have intercepts f, g upon the coordinate axes, prove that

$$\frac{a^2}{f^2} + \frac{b^2}{g^2} = 1.$$

14. Investigate the condition that $y = mx, y = m'x$ may be conjugate diameters of the hyperbola $b^2x^2 - a^2y^2 = a^2b^2$.

Find the equation of that diameter of the hyperbola $2x^2 + xy - y^2 = 9a^2$, which is conjugate to the diameter coinciding with the axis of x , and prove that the difference of the squares of these two conjugate diameters is equal to $16a^2$.

VI. MECHANICS.

[Full marks may be obtained for about two-thirds of this paper. Great importance is attached to accuracy. N.B.— g may be taken = 32.]

1. Two equal weightless rods, AB and AC , are hinged together at A at an angle of 74° , and placed in a vertical plane, with B and C on a smooth horizontal plane. If B and C be connected by a string, and a weight of 30 lbs. be suspended at D , where $AD : DC = 5 : 4$, find graphically the tension of the string.

2. Four equal weightless rods are hinged together to form the rhombus $ABCD$, and the hinges A and C are connected by a string. If the rhombus be suspended from A , and equal weights of 1 cwt. each be suspended from B and D , find graphically the tension of the string.

3. Four equal weightless rods are hinged together to form the rhombus $ABCD$, and the hinges at B and D are joined by the weightless rod BD . The angle $BAD = 64^\circ$. If the rhombus be suspended from A , and a weight of 1 cwt. be suspended from C , find the thrust in BD .

4. Prove that the centre of gravity of a triangle coincides with that of three equal particles at the angles.

The uniform plane quadrilateral $ABCD$ right-angled at A and obtuse-angled at B is bisected by the diagonal AC . If it be placed with its plane vertical and the side AB on a horizontal plane, prove that the condition of equilibrium is $2BM < 3AB$, where M is the foot of the vertical from C on AB produced.

5. Explain the construction and action of the screw, and find the relation of the power to the weight when there is no friction.

6. A uniform beam, weight W , laid on a horizontal plane, can be just moved by pushing it with a horizontal force $\frac{1}{3} W\sqrt{3}$. Prove that the least force which can move it is equal to $\frac{W}{2}$, and find the direction of this force.

If the beam be pulled slowly by a rope attached at one end A , prove that A will not rise from the ground unless the inclination of the rope to the horizontal be 60° at least.

7. A body is projected from a given point O , with the velocity v , at the angle α to the horizon; find its distance from O after the time t .

If any number of such particles be projected from O with the same velocity in different directions, prove that at any subsequent instant they will all lie on the surface of a sphere.

8. Find the horizontal and vertical accelerations of a body sliding down a smooth inclined plane.

If two bodies start at the same instant sliding down two lines in the same vertical plane sloping towards the same direction, at angles α and β to the horizon, prove that each as seen from the other will always appear to be moving parallel to a line inclined to the horizon at the angle $\alpha + \beta$.

9. Find the velocities of two elastic spheres of given masses and elasticities after direct impact.

If two perfectly elastic smooth spheres, A and B , impinge upon each other directly or obliquely, prove that B 's velocity relative to A , after impact, will make the same angle with the line of centres as the relative velocity of A to B did before impact.

10. Find the acceleration of a particle describing a circle, radius a , with velocity v .

A circus horse gallops round a circle of 30 feet radius, at a speed of 15 miles an hour, prove that the least value of the coefficient of friction between feet and ground, that the horse may not slip, neglecting the distance between the feet and centre of gravity, is $\frac{1}{2}$ very nearly.

11. A train weighs M tons, and the resistance of friction is p lbs. per ton. If the engine can exert a pull of P lbs., and the break a resistance of R lbs., find the distances passed over in attaining a speed of v miles per hour from rest, and in slowing down from that speed to rest respectively moving on the level.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1893.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.),

[*Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Define *right angle*, *trapezium*, *quadrilateral*.

If two triangles have two sides of the one equal to two sides of the other, each to each, but the angle contained by the two sides of one of them greater than the angle contained by the two sides equal to them of the other, the base of that which has the greater angle shall be greater than the base of the other.

$ABCD$, $EFGH$ are quadrilaterals in which the sides AB , BC , EF , FG are all equal to one another; and the angles at C , D , G , H are right angles. Show that if the angle at F is greater than that at B , then is EH greater than AD .

2. What is the magnitude of the angle of a regular figure of n sides?

Three regular figures, of n_1 , n_2 , and n_3 sides, have one vertex in common; and just fill the whole space at that vertex; show that

$$\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} = \frac{1}{2}.$$

WOOLWICH ENTRANCE EXAMINATION.

3. Describe a square on a given straight line. What does the demonstration indicate concerning a parallelogram which has a right angle?

4. If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the two parts.

What is the equivalent of this in algebraical symbols?

5. If in the triangle ABC , which has the obtuse angle ACB , AD be drawn perpendicular to BC produced, the square on AB is greater than the squares on AC and CB by twice the rectangle BC, CD . Prove this, and give the value for the square on AC in terms of the sum of the squares on AB and BC and a rectangle under segments of the base.

In a quadrilateral $ABCD$, AC is equal to CD , AD is equal to BC , and the angle ACB is supplementary to the angle ADC ; show that the square on AB is equal to the sum of the squares on BC, CD, DA .

6. If two circles touch one another externally, the straight line which joins their centres shall pass through the point of contact.

7. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

ABC is an isosceles triangle in which AB is equal to AC ; and AD is drawn to meet the base BC in D ; show that the centre of the circle described round ABD is at the same distance from AB that the centre of the circle round ADC is from AC .

8. Define *tangent*, *similar segments of circles*.

If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle.

9. When are rectilinear figures said to be *inscribed in* and *described about* a circle?

About a given circle describe a triangle equiangular to a given triangle.

10. Inscribe an equilateral and equiangular hexagon in a given circle.

Show that the regular hexagon inscribed in a circle is three-fourths of the regular hexagon described about the same circle.

11. If the sides of two triangles, about each of their angles, be proportionals, the triangles shall be equiangular to one another and shall have those angles equal which are opposite to the homologous sides.

12. Similar triangles are to one another in the duplicate ratio of their homologous sides.

ABC is a triangle with an obtuse angle at A ; and AM , AN are drawn to meet the base BC in M and N respectively, so that the angles AMB , ANC are each equal to BAC ; show that BM has to NC the duplicate ratio of that which AB has to AC .

II. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—Great importance will be attached to accuracy.]

1. Resolve into real elementary factors

$$6x^2 - 23xy + 20y^2, \quad x^4 - 7x^2y^2 + y^4, \quad x^6 - 1.$$

2. Divide $(x+y)^5 - x^5 - y^5$ by $x^2 + xy + y^2$.

3. Find the Highest Common Factor of the expressions

$$x^4 - 3x^3 + 10x^2 - 11x + 15 \quad \text{and} \quad 3x^4 - 7x^3 + 15x^2 - x - 10.$$

4. Simplify

$$(i.) \quad \frac{9}{x^2 - x - 20} - \frac{7}{x^2 + x - 12} - \frac{2}{x^2 - 8x + 15},$$

$$(ii.) \quad \frac{\frac{1}{x-a} - \frac{1}{x+a} - \frac{2a}{x^2+a^2}}{\frac{1}{x^3-a^3} - \frac{1}{x^3+a^3}} \left(\frac{1}{x^2+ax+a^2} + \frac{1}{x^2-ax+a^2} \right),$$

$$(iii.) \quad \frac{\sqrt{3}}{\sqrt{3}+\sqrt{5}+\sqrt{7}} - \frac{\sqrt{3}}{\sqrt{5}+\sqrt{7}-\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{7}+\sqrt{3}-\sqrt{5}} + \frac{\sqrt{3}}{\sqrt{3}+\sqrt{5}-\sqrt{7}}.$$

5. Obtain the square root of each of the expressions :

$$(i.) \quad 9x^4 - 42x^3 + 37x^2 + 28x + 4,$$

$$(ii.) \quad 9 - \sqrt{77}.$$

6. Prove that the form of the expression

$$\frac{(x^4 - 4x^3 + 8x - 4)^2}{(x^2 - 2x + 2)^4}$$

is unaltered by the substitution of $\frac{2}{2-x}$ for x .

7. Solve the equations

$$(i.) \quad \frac{3x+5}{13} - \frac{5x-2}{11} = \frac{4x-3}{5} - \frac{5x+1}{6},$$

$$(ii.) \quad \sqrt{x+2} + \sqrt{x-3} = \sqrt{3x+4},$$

$$(iii.) \quad x^2 + xy + y^2 = 19, \quad x^4 + x^2y^2 + y^4 = 133.$$

8. If α and β be the roots of the quadratic equation

$$ax^2 + bx + c = 0,$$

prove that

$$\alpha + \beta = -\frac{b}{a}, \text{ and } \alpha\beta = \frac{c}{a};$$

and also construct the quadratic whose roots are

$$\frac{1-\alpha}{1+\alpha} \text{ and } \frac{1-\beta}{1+\beta}.$$

9. The sum of five numbers in arithmetical progression is 10, and the sum of their squares is 60; find the numbers.

10. Prove that if

$$ma^3 + nc^3 : pa^3 + qc^3 :: mb^3 + nd^3 : pb^3 + qd^3,$$

then either

$$a : b :: c : d \text{ or } m : n :: p : q.$$

11. Write down the first negative term in the expansion of $(1+x)^{\frac{27}{5}}$ with the aid of the Binomial Theorem, taking x to be a positive quantity.

12. Explain how to deduce the value of the logarithm of a number to any given base, from the logarithm of that number to base 10.

Find, with the aid of the logarithmic tables supplied, an approximate value for the incommensurable root of the equation

$$3^{2x} - 5 \times 3^x + 6 = 0.$$

III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy.]

1. Define the circular measure of an angle, and, assuming that $\pi = \frac{22}{7}$, find, to three places of decimals, the number of degrees in the unit of circular measure.

Also find, to three places of decimals, the number of degrees in the angle whose circular measure is $\cdot 63$.

2. Write down the expressions for $\sin(A+B)$ and $\cos(A+B)$ in terms of the sines and cosines of A and B , and prove the formula

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

Hence prove that

$$\tan 15^\circ = 2 - \sqrt{3} \quad \text{and} \quad \tan 75^\circ = 2 + \sqrt{3}.$$

3. If $\sin A = \frac{3}{17}$, find the values of $\sin 2A$, $\cos 2A$, and $\tan 2A$.

4. Prove the formulæ

$$(i.) \quad \operatorname{cosec} 2A = \cot A - \cot 2A.$$

$$(ii.) \quad \sin^2\left(\frac{\pi}{8} + \frac{\theta}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{\theta}{2}\right) = \frac{1}{\sqrt{2}} \sin \theta.$$

$$(iii.) \quad \frac{\sin 3A + \sin 4A + \sin 5A}{\cos 3A + \cos 4A + \cos 5A} = \tan 4A.$$

5. Find an expression for all the angles which have the same tangent as a given angle.

$$\text{If} \quad \frac{1 + \tan \theta}{1 - \tan \theta} = 3 \frac{1 - \tan \theta}{1 + \tan \theta},$$

prove that

$$\theta = n\pi + \frac{\pi}{12} \quad \text{or} \quad n\pi + \frac{5\pi}{12},$$

where n is any integer.

6. If A, B, C are the angles of a triangle, prove the formulæ

$$(i.) \quad \sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}.$$

$$(ii.) \quad \cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}.$$

7. A tower stands on a horizontal plane, and it is observed that at a position on the plane, distant 237 feet from the foot of the tower, the angle of elevation of the top of the tower is $37^{\circ} 15' 15''$; find, to two places of decimals, the height of the tower.

8. Prove that, for a triangle ABC ,

$$\sin \frac{A}{2} = \left\{ \frac{(s-b)(s-c)}{bc} \right\}^{\frac{1}{2}}.$$

Having given that

$$a = 145, \quad b = 235, \quad c = 200,$$

calculate the angle A .

9. Find expressions for the radii of the inscribed and escribed circles of a triangle ABC .

If r , r_1 , r_2 , r_3 , represent these radii, prove that

$$\frac{rr_1}{r_2r_3} = \tan^2 \frac{A}{2}.$$

10. A spectator on one side of a straight road is looking in the direction perpendicular to the direction of the road, and observes that a telegraph post is exactly opposite and that α is the angular elevation of the top of this post. He then turns round and observes that β is the angular elevation of the next post. Assuming that the height of each post is the same, and that c is the distance between the posts, prove that the height of each is equal to

$$c \tan \alpha \tan \beta \div \sqrt{\tan^2 \alpha - \tan^2 \beta}.$$

11. Two cylindrical vessels are filled with water; the radius of one vessel is six inches and its height one foot, and the radius of the other is eight inches and its height one foot and a half; find the radius of a cylindrical vessel eleven inches in height which will just contain the water in the two vessels.

12. Having given that the length of each edge of a regular tetrahedron is four inches, determine, to three places of decimals, the number of square inches in the total surface of the tetrahedron.

Also find the number of cubic inches in the volume of the tetrahedron.

IV. STATICS AND DYNAMICS.

1. Enunciate and prove the proposition known as the "Polygon of Forces."

OA , OB , OC are three straight lines inclined at angles of 120° to one another; a force of $3P$ acts from A to O , a force of $4P$ from O to B and a force of $5P$ from O to C . Show by a carefully drawn figure how you would graphically determine the magnitude and direction of the resultant of the three forces.

2. B , C are two smooth rings fixed in space at a distance apart = 25 inches, B being 9 inches, and C 16 inches above the ground. A string $ABCD$ passes through the rings and supports equal weights W , W at its extremities A , D . Find the resultant pressures of the string upon the rings.

3. Two parallel forces $5P$, $7P$ act at points A , B respectively. State clearly the position of the line of action and the magnitude of their resultant (i.) when the forces are like, and (ii.) when unlike.

A horizontal bar AB , 7 feet long, is supported at its extremities, and a man of 150 lbs. weight hangs from it by his hands, one being 1 foot from A , the other 3 feet from B . Find the pressures on the supports due to the weight of the man.

4. Three forces (not parallel) acting in one plane upon a rigid body are in equilibrium. Show that if any triangle be formed by drawing straight lines perpendicular to the directions of the forces, its sides will be proportional to the forces.

A triangle ABC (whose weight may be neglected) rests in a vertical plane with the middle points of the sides AB , AC in contact with two smooth pegs, the line joining them being horizontal and parallel to the base BC . Determine graphically, or otherwise, the point in BC where a weight W may be placed without disturbing the equilibrium; and if $W = 10$ lbs., and AB , AC and BC be 4, 5 and 6 feet respectively, find the pressures on the pegs.

5. Show that if a body be placed on a horizontal plane it will stand or fall according as the vertical line through its centre of gravity falls within or without the base. Explain what is meant by the "base."

Why does a bicycle rider when on the point of falling over to one side steer his machine to that side?

6. Distinguish between the three kinds of levers and give examples of each.

Two levers OA , OB of lengths 3 and 4 feet respectively can turn in a vertical plane about a common fulcrum O , and their middle points are connected by a string whose length is $2\frac{1}{2}$ feet. Find the least force which applied at A will keep OB horizontal with a weight of 12 lbs. suspended from B . Find also the tension of the string.

7. "Suppose mud composed of coarser particles to fall at the rate of two feet per hour, and these to be discharged into that part of the Gulf-stream which preserves a mean velocity of three miles an hour for a distance of two thousand miles; in twenty-eight days these particles will be carried x miles, and will have fallen only to a depth of y fathoms." (Lyell's *Principles of Geology*.)

Find the numbers given for x and y in this passage.

8. Assuming that the space described from rest in time t by a particle moving with uniform acceleration α is equal to $\frac{1}{2}at^2$, deduce the corresponding expression for the case when the particle has an initial velocity u .

If the space described in the fifth and sixth seconds from rest be 25 feet, find the acceleration.

9. Distinguish between (i.) acceleration and accelerating force, and (ii.) the mass and weight of a body; and investigate an expression for the acceleration of a particle sliding down a smooth inclined plane under the action of gravity.

Two particles start simultaneously from rest, the one down an inclined plane AC of length 25 feet, the other down a plane BC of length 70 feet, the heights of A , B above the horizontal plane through C being 7 and 56 feet respectively. Find which particle will arrive at C first, and when at C how far it will be from the other particle.

10. Two bodies whose masses are P , Q are connected by a string which passes round a smooth pulley; find the acceleration.

WOOLWICH ENTRANCE EXAMINATION.

Show that, if $\frac{3}{4}W$ be added to one of the weights in question (2), and $\frac{3}{4}W$ be taken from the other, the pressures on the rings will be unaltered.

11. A particle is projected at an angle of 45° to the horizon with the velocity it would acquire in falling freely for one second ; show in a figure the position of the particle at the end of the first, second, and third seconds of the motion ; and taking any unit of length to represent the initial velocity, mark against each straight line of your diagram the measure of its length.

FURTHER EXAMINATION.

V. PURE MATHEMATICS.

[Full marks may be obtained for about three-fifths of this paper. No Candidate must attempt more than ten questions. Great importance will be attached to accuracy.]

1. A moveable circle with constant radius cuts the two fixed straight lines APR , AQS in P , Q , R , and S , prove that for all positions of the circle the sum of the arcs RPQ and PQS is constant.

2. Factorise the expression

$$1 + y(1 + x)^2(1 + xy).$$

3. Solve the equation

$$x^2 + px + q = 0.$$

If the roots of $x^2 + Px + Q = 0$ be α and β , and the roots of $x^2 + px + Q = 0$ be γ and δ , find the roots of $x^2 + px + q = 0$ in terms of α , β , γ , and δ .

4. Find the number of combinations of n things r together.

There are $2n$ letters, of which 2 are a , 2 are b , and so on: find how many different algebraical products can be formed, in each of which the sum of the indices of the letters is 3.

5. Prove that the sum of the squares of the coefficients of $(1 + x)^n$ is

$$\frac{\overline{2n}}{\overline{n} \overline{n}},$$

and find the sum of the products taken two and two together.

6. Expand $\log_e(1 + x)$ in ascending powers of x .

Prove that the coefficient of x^n in the expansion of

$$\log_e(1 + x + x^2 + \dots + x^{m-1})$$

is either

$$-\frac{m-1}{n} \quad \text{or} \quad \frac{1}{n}$$

according as n is, or is not, a multiple of m .

7. Find the third side of a triangle in terms of the two other sides and the included angle.

Apply the result, or any other method, to prove that if α and β be each less than π , and x be positive,

$$\sqrt{1 - 2x \cos \alpha + x^2} + \sqrt{1 - 2x \cos (\beta - \alpha) + x^2} > 2 \sin \frac{\beta}{2}.$$

8. Find the radii of the three escribed circles of a triangle.

If these circles be discs, fixed in position, and be enveloped by a string drawn tight round them, prove that the sum of the straight portions of the string will be equal to twice the perimeter of the triangle.

9. Find the value of

$$\sin \alpha + x \sin 2\alpha + x^2 \sin 3\alpha \text{ ad infin.}; x \text{ being } < 1.$$

Prove that

$$\sin \alpha + x \sin 2\alpha + x^2 \sin 3\alpha + \text{etc.} = \sin \alpha + y \sin 2\alpha + y^2 \sin 3\alpha + \text{etc.}$$

if x and y are proper fractions satisfying the condition $x + y = 2 \cos \alpha$.

10. From the given point O , any line OP is drawn meeting a given straight line AB in P , and through P , a line PQ is drawn at a given angle α to OP . Prove that for all directions of OP the line PQ will be a tangent to a certain parabola and find the focus and directrix of this parabola.

11. In the hyperbola prove that $SP - HP$ is equal to the transverse axis.

If circles be described with centres S and H , the difference of whose radii is equal to the transverse axis, and from P any point on the branch of the hyperbola nearer to the larger circle, PS and PH be drawn and produced to meet the circles in Q and R respectively, prove that the locus of the intersection of the tangents to the circles at Q and R will be a straight line.

Enunciate and prove the corresponding proposition when P is on the other branch.

12. Prove that $Ax + By = C$ is the equation of a straight line.

Interpret the constants when

$$(i.) A = 1. \quad (ii.) B = 1. \quad (iii.) C = 1. \quad (iv.) A^2 + B^2 = 1.$$

13. Find the intercepts on the axes made by the normal at any point x', y' on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Hence prove that if M and N be the feet of the perpendiculars upon the axes from any point P of an ellipse, the line MN will be always normal to a concentric and similar ellipse.

14. Find the equations of the two straight lines conjugate to the coordinate axes of x and y respectively in the curve

$$Ax^2 + 2Bxy + Cy^2 = 1.$$

Find the condition that these straight lines should coincide, and interpret the result.

VI. MECHANICS.

[Full marks may be obtained for about two-thirds of this paper. Great importance is attached to accuracy. N.B.— g may be taken = 32.]

1. Forces 1, 2, 4, 4 lbs. respectively, act along the sides AB , BC , CD , DA of a square. Find by graphical construction the magnitude and direction, and point of application in the line BC , of the force which would balance the system.

2. $ABCD$ is a rhombus formed by four weightless rods loosely jointed together, and the figure is stiffened by a weightless rod, of half the length of each, jointed to the middle points of AB , AD . If this frame is suspended from A , and a weight of 100 lbs. attached to it at C , find by graphical construction the thrust of the cross rod.

3. A , B are two fixed pegs, B being at a higher level than A , and a heavy rod rests on B and passes under A ; show graphically that, the angle of friction between the rod and the pegs being the same in both, the rod will rest in any position in which its centre of gravity is beyond B , provided the inclination of AB to the horizon is less than the angle of friction; and for any greater inclination determine graphically the limiting distance of the centre of gravity beyond B consistent with equilibrium.

Verify your result from the equations of equilibrium.

4. Find the distance of the centre of gravity of a number of heavy particles in the same straight line from a fixed point in the line.

How many like coins, having diameters 20 times the thickness, can be piled on a table so that their centres may be in a straight line inclined at an angle of 45° to the horizon?

5. Roberval's balance consists of a parallelogram $AA'B'B$, loosely jointed at the angles, whose opposite sides AA' , BB' , are moveable about smooth pivots through their middle points fixed in a vertical line, and with

arms CD , $C'D'$ rigidly attached at right angles to AB , $A'B'$ respectively. Prove (by the principle of work or otherwise) that a weight suspended from any point in CD will balance an equal weight suspended from any point in $C'D'$, in any position of the system. Determine the horizontal stresses at the angular points of the parallelogram. Can you determine the vertical stresses at the same points?

6. The trail of smoke from a steamer on a course due north is observed to extend in the direction E.S.E., while that from another, on a course due south, with the same uniform speed, is observed to be N.N.E.; determine the speed and direction of the wind.

7. A steamer starts from a pier, steering eastward with uniform acceleration, the wind blowing steadily from the south. Show that the line of trail of its smoke is an arc of a parabola, with a fixed directrix and constant latus rectum.

8. Prove that the velocity at any point in the path of a projectile (*in vacuo*) is that due to its distance from the directrix.

If AC , BC bisect the angles between AB and the verticals through A , B respectively, and if F is the foot of the perpendicular from C on AB , the least velocity of projection which will carry a projectile from A to B is that due to a height equal to AF .

9. Show that, if a point moves in a curve with constant speed, its acceleration at any point is in the normal at that point, and determine the acceleration when the curve is a circle.

A mass of 1 lb. is suspended by a string 2 feet long in a railway carriage. Show that when the train is moving round a curve, whose radius is 22 chains, at the rate of 30 miles an hour, the tension of the string is increased by about $\frac{1}{2}$ oz., and the horizontal displacement of the weight is 1 in. nearly.

10. Define a *foot-pound* and a *horse-power*.

A locomotive engine, which can work up to 100 horse-power, is attached to a train, whose mass (including the locomotive itself) is 100 tons; assuming the total resistance to be constant and equivalent to 10 lbs. weight per ton, find the greatest speed of the train in miles per hour. When travelling at this speed the steam is shut off; find the distance and the time in which the train would be reduced to rest by the resistance alone.

11. Prove that the kinetic energy of a system of two masses m_1, m_2 , moving with the speeds u_1, u_2 in the same straight line, is equal to

$$\frac{1}{2}(m_1 + m_2)\bar{u}^2 + \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2,$$

where $\bar{u}$ is the speed of their centre of mass; and hence, that the kinetic energy lost by their collision is equal to

$$\frac{1}{2} \cdot \frac{(1 - e^2)m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2$$

where e is the coefficient of resilience.

Account generally for the energy said to be "lost."

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1893.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.]

1. Any two sides of a triangle are together greater than the third side.

ABC is any rectilineal angle less than the angle of an equilateral triangle, and D and E are two points within it; find the points F and G in AB and BC respectively, such that the sum of the lines DF , FG , GE has the least possible value.

2. Equal triangles on the same base and on the same side of it are between the same parallels.

Use this proposition to show that the straight line joining the middle points of two sides of a triangle is parallel to the third side.

3. Describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given rectilineal angle.

4. If a straight line be divided into two equal, and also into two unequal parts, the squares on the two unequal parts are together double of the square on half the line and of the square on the line between the points of section.

Express the proposition as an algebraical formula.

5. Draw a straight line from a given point, either without or in the circumference, which shall touch a given circle.

From a given point P without the circle ABC , draw a straight line PAB cutting the circle in A and B so that AB may be equal to a given straight line D .

6. Distinguish between the angle of a segment and the angle in a segment and define *similar segments of circles*.

$ABCD \dots$ is a straight line, on parts of which $AB, AC, AD, \dots$ similar segments of circles $AFB, AGC, AHD, \dots$ are described; show that (1) all these circles have a common tangent at A , (2) any straight line $AFGH \dots$ drawn from A and cutting the circles in $F, G, H, \dots$ cuts off similar segments.

7. On a given straight line describe a segment of a circle containing an angle equal to a given rectilineal angle.

Given the altitude, the vertical angle, and the perimeter of a triangle, construct it.

8. Describe a circle about a given triangle.

Point out and prove any facts concerning the opposite angles or sides of a quadrilateral (1) inscribed in, (2) described about, a circle.

9. When is the first of four magnitudes said to have the same ratio to the second that the third has to the fourth?

Illustrate this definition by giving Euclid's proof of the proposition that triangles of the same altitude are to one another as their bases.

10. Parallelograms which are equiangular to one another have to one another the ratio which is compounded of the ratios of their sides.

11. In any right-angled triangle, any rectilineal figure described on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle.

If BFC , CDA , AEB be equilateral triangles described externally on the sides of a triangle ABC right-angled at A ; and if AG be drawn perpendicular to BC to meet BC in G ; show that the triangles BFG , CFG are respectively equal to the triangles AEB , CDA .

II. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—Great importance will be attached to accuracy.]

1. Divide $(x+y)^4 + (x^2-y^2)^2 + (x-y)^4$ by $3x^2+y^2$.
2. Resolve each of the following into three real factors :
 $4x^3 - 23x^2 + 28x$, $y^4 + 11y^2 - 180$, $a^6 + 27b^6$.
3. Find the Highest Common Factor of
 $5x^4 - 16x^3 + 20x + 7$ and $30x^3 - 71x^2 + 49$.

4. Simplify

- (i.) $\frac{1}{x^2-4x+3} - \frac{4}{x^2+2x-15} + \frac{3}{x^2+4x-5}$;
- (ii.) $\left\{ \frac{a^3-b^3}{b^3-a^3} - 3\left(\frac{a^2}{b^2} + \frac{b^2}{a^2}\right) + 5 \right\} \div \left(\frac{a}{b} - 1 - \frac{b}{a}\right)^2$;
- (iii.) $\left\{ \frac{\sqrt{3}}{\sqrt{5}+\sqrt{2}} - \frac{\sqrt{3}}{\sqrt{500}-\sqrt{200}} \right\} \div \left\{ \frac{5}{\sqrt{15}+\sqrt{6}} - \frac{1}{\sqrt{60}-\sqrt{24}} \right\}$.

5. Find the square root of

- (i.) $4x^4 + 12x^3 - 11x^2 - 30x + 25$;
- (ii.) $41 - \sqrt{720}$.

6. Solve the equations :

- (i.) $ax - by = a^2$, $bx - ay = b^2$;
- (ii.) $\frac{7x}{1 - \frac{2x-12}{3x-5}} = \frac{48}{1 - \frac{1}{x}}$.

Divide 1 into two fractions such that the sum of their cubes is $\frac{1}{8}$.

7. When are the roots of $ax^2 + bx + c = 0$ real?

If x is real, show that $\frac{x^2 - 4x - 20}{x - 7}$ cannot have a real value between 8 and 12.

8. Show that a ratio of greater inequality is diminished by adding any, the same, quantity to both terms of the ratio.

9. If b is half the Harmonic Mean between a and c , then

$$a^3 - b^3 + c^3 + 3abc = (a - b + c)^3.$$

10. Prove by *Mathematical Induction* that, when n is a positive integer, $(3n + 1)7^n - 1$ is always divisible by 9.

11. When $x = \frac{1}{3}$, find the two greatest terms in the expansion of

$$(1 - x)^{-13}.$$

12. Prove that

$$\log_y x \times \log_x y \times \log_x z = 1.$$

Show, with the aid of the logarithmic tables supplied, that approximately

$$200^{\frac{1}{10}} + \log_{10} 200149 = 1.$$

III. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—*Great importance will be attached to accuracy.*]

1. If the radius of a circle be 25 feet, find, to three places of decimals, the length of the arc subtending an angle of 3° at the centre.

$$\pi = 3.1416.$$

2. Find $\tan \theta$ in terms of $\cos \theta$, and $\tan(\phi + \theta)$ in terms of $\tan \phi$ and $\tan \theta$.

If $\cos \theta = \frac{3}{5}$ and $\phi - \theta = \frac{\pi}{4}$, find $\tan \phi$ and $\tan(\phi + \theta)$.

3. Prove that

$$\sin A + \sin B = 2 \sin \frac{(A+B)}{2} \cos \frac{(A-B)}{2},$$

and express in a single term

$$(i.) \quad \sin A \sin 2A + 2 \cos A \cos 2A.$$

$$(ii.) \quad \frac{1}{\sqrt{2} \cot \frac{\theta}{2} - \operatorname{cosec} \frac{\theta}{2}} + \frac{1}{\sqrt{2} \cot \frac{\theta}{2} + \operatorname{cosec} \frac{\theta}{2}}.$$

4. If A and B are two angles, each positive, and less than 90° and such that

$$3 \sin^2 A + 2 \sin^2 B = 1,$$

$$3 \sin 2A - 2 \sin 2B = 0.$$

prove that

$$A + 2B = 90^\circ.$$

5. Prove the following identities :

$$(i.) \quad 2 \operatorname{cosec} 4A + 2 \cot 4A = \cot A - \tan A.$$

$$(ii.) \quad (1 + \sec \theta + \tan \theta)(1 + \operatorname{cosec} \theta + \cot \theta) \\ = 2(1 + \tan \theta + \cot \theta + \sec \theta + \operatorname{cosec} \theta).$$

6. Find the cosines of the angles of a triangle in terms of the sides.

If the sides of a triangle are in arithmetical progression, prove that the middle angle must be less than 60° .

7. If one side of a triangle be equal to m times the difference of the other two, prove that the cosine of half the angle subtended by that side will be m times the sine of half the difference of the remaining angles.

8. If in a triangle $a = 3$, $b = 1$, $C = 53^\circ 7' 48''$, find c .

9. What is meant by the ambiguous case in the solution of plane triangles and when does it occur?

Prove that the sum of the areas of the two triangles satisfying the given conditions is known, and find it in terms of given quantities.

10. A building on a square base $ABCD$ has the sides of the base AB and CD , parallel to the banks of a river. An observer standing on the bank of the river furthest from the building in the same straight line as DA finds that the side AB subtends at his eye an angle of 45° , and after walking a yards along the bank he finds that DA subtends the angle whose sine is $\frac{1}{3}$.

Prove that the length of each side of the base in yards is $\frac{1}{2}\sqrt{2a}$.

11. A cylindrical boiler, terminated by plane ends, is internally 15 feet long and 4 feet in diameter and is traversed lengthwise by 50 cylindrical fire tubes, each 3 inches in external diameter, determine the volume of water the cylinder could contain, taking π to be $\frac{22}{7}$.

WOOLWICH ENTRANCE EXAMINATION.

12. Supposing an ice field to exist round one of the earth's poles, extending 5° from the pole in all directions, find the area of the ice field in square miles, taking the earth's radius to be 4000 miles, $\cos 5^\circ$ to be .996195 and $\pi = \frac{22}{7}$.

IV. STATICS AND DYNAMICS.

[N.B.—*The acceleration due to gravity is to be taken = 32.*]

1. Enunciate the proposition known as the “Parallelogram of Forces,” and assuming it for the direction, prove it for the magnitude, of the resultant.

Four forces in equilibrium acting at a point are represented in magnitude and direction by AB , CD , AD , CB . Show that A , B , C , D must be the angular points of a parallelogram.

2. Given the magnitudes of the forces acting in one plane on a particle and the angles which their directions make with a given straight line in the plane, find the magnitude of their resultant and the angle which its direction makes with the given line.

Forces of 3, 4 and 6 lbs. make angles of 90° , 60° , and 30° respectively with a force of 2 lbs. (the angles being measured in the same direction). Show that the resultant of the four forces is equal to $(8 + 3\sqrt{3})$ lbs.; and find (with the aid of the logarithmic tables supplied) the angle its direction makes with the force of 2 lbs.

3. Define the centre of gravity of a rigid body, and show how its position may be experimentally determined.

A lamina hangs freely in a vertical plane with a point A fixed, and in this position a horizontal line BC is drawn upon it; it next hangs with B fixed and a horizontal line through A is drawn cutting BC in C ; show that, if C be fixed, the lamina will rest with the line joining A , B horizontal.

4. State clearly the conditions of equilibrium of a system of forces acting in one plane upon a rigid body.

WOOLWICH ENTRANCE EXAMINATION.

A square board $ABCD$ is placed upon a smooth horizontal table and a given force P acts from E , the middle point of AB , towards F , the middle point of CD . Find the magnitudes of three forces X , Y , Z which, acting along BC , CA , AB respectively, will make equilibrium with P .

5. Investigate the relation between the power and the weight in that system of pulleys in which the same string passes round all the pulleys.

Draw a diagram of the case in which there are three pulleys at each block.

6. Define *velocity*, and show how it is measured (i.) when uniform and (ii.) when variable.

A train is moving at the rate of 40 miles an hour, what would be the measure of its velocity if a *yard* and a *second* were the units of space and time?

7. Investigate an expression for the space described from rest in time t by a point moving with acceleration a .

What would be the expression for the space if the point started with a velocity u ?

A particle is projected vertically upwards with a velocity of 50 feet per second, when will it be at a height of 25 feet above the point of projection? Also, where will it be when it is moving with a velocity of 25 feet per second?

8. Explain the meanings of the symbols in the equation $F = Ma$, and having given that the units of space and time are one foot and one second, state the relation implied between the other units involved.

Find the force which acting on a mass of 8 ounces increases its velocity by 4 feet a minute in every second; and state clearly the unit in terms of which your answer is expressed.

9. A string attached to a body of mass P placed on a smooth horizontal table passes over one of the edges and has a body of mass Q attached to its other extremity. Find the acceleration of the system.

If Q be divided into two equal parts one hanging below the other by a separate string, find the tension of this string.

10. A particle is projected in a given direction with a given velocity, show how to find its position and the direction in which it is moving at the end of a given time. .

A stone is thrown from the top of a tower with a velocity of g feet per second in a direction making an angle α with a line drawn vertically upwards through the point of projection; prove that at the end of two seconds the line joining the stone to the point of projection will make an angle of $\frac{1}{2}(\pi + \alpha)$ with the vertical line.

FURTHER EXAMINATION.

V. PURE MATHEMATICS.

[Full marks may be obtained for about three-fifths of this paper. No Candidate must attempt more than ten questions. Great importance will be attached to accuracy.]

1. Describe a circle touching three given straight lines.

If a circle be inscribed in an isosceles right-angled triangle, show that its diameter is equal to the excess of the sum of the equal sides over the hypotenuse.

2. Under what circumstances will the solution of two simultaneous equations in two unknown quantities depend upon the solution of a quadratic equation?

Two trains are proceeding in the same direction upon the same line of rails, each with uniform speed. The quicker train, which is in front, gains $\frac{1}{n}$ of a mile every minute, and also $\frac{1}{n}$ of a minute every mile upon the slower train. Determine the speed of each train in miles per hour, and show that, n being a positive quantity, the speed of the quicker train cannot be less than 60 miles an hour.

3. Find the sum of n terms of a geometrical progression. Sum to n terms, and also to infinity, the series

$$(1+x^n)^{-\frac{1}{n}} + x(1+x^n)^{-\frac{2}{n}} + x^2(1+x^n)^{-\frac{3}{n}} + \dots$$

4. State the circumstances under which

$$(1+x)^n$$

is expressible in the form of a convergent infinite series.

Show how to expand

$$(1+yz+zx+xy)^{-1}$$

by the binomial theorem, and show that the coefficient of x^2y^2z is equal to the coefficient of the same term in the development of the product

$$(y-z)^3(z-x)^2(x-y).$$

5. What is the characteristic of the logarithm of a number? Prove the rule for assigning the characteristic of a given decimal number.

Expand $\log_a x$ in a series proceeding by ascending powers of

$$\frac{x-1}{x+1}.$$

6. One angle of a triangle is given, and the ratio of the sines of the other angles. Show how to find these angles by a formula adapted to logarithmic computation.

Find the angles B, C from the data

$$A = 60^\circ, \quad \frac{\sin B}{\sin C} = 2.$$

7. Express the area of a triangle in terms of its sides. Show that the formula involves Euclid's theorem that two sides of a triangle are together greater than the third side.

Find an expression for the area in terms of the sums of first, second, and third powers of the sides.

If the sum of the sides is 12 feet, and the sum of the squares on the sides is 50 square feet, and the sum of the cubes on the sides is 216 cubic feet, find the area of the triangle.

8. Find expressions for the curved surface and volume of a right circular cone.

A conical tent is five feet high. Find the radius of its base so that the number of square feet of canvas may be equal to the number of cubic feet of space inside the tent.

9. Show how to transform from rectangular coordinates to oblique, and from oblique to rectangular.

The equation of a straight line referred to axes inclined at 30° is

$$y = 2x + 1.$$

Find its equation referred to axes inclined at 45° , the origin and axis of x remaining unchanged.

10. Define an ellipse, and prove geometrically that the sum of the focal radii of any point upon it is equal to the major axis.

11. Points $(1, 0)$, $(2, 0)$, are taken on the axis of x , the axes being rectangular. On the line connecting the points an equilateral triangle is described so that the coordinates of its vertex are both positive. Find the equations of the circles described upon its sides as diameters.

12. Find the equation to the tangent at any point of an ellipse, and the lengths of its intercepts upon the coordinate axes.

In an ellipse of eccentricity $\tan \beta$, a focal chord is inclined at an angle α to the major axis; show that the tangents at the extremities of the chord include an angle

$$\tan^{-1}(\tan 2\beta \sin \alpha).$$

13. If a small portion of a hyperbola be given, show how to verify its hyperbolic nature geometrically.

If two sides of a triangle be given in position, and its perimeter given in magnitude, show that the locus of the point which divides the base in a given ratio is a hyperbola.

VI. MECHANICS.

[Full marks may be obtained for about two-thirds of this paper. Great importance is attached to accuracy. N.B.— g may be taken = 32].

1. Forces are represented in magnitude, direction and position, by the sides of a polygon taken in order; prove that they are equivalent to a single couple, and that the magnitude of the couple is proportional to the area of the polygon.

2. A uniform beam, 12 feet in length, has a fixed hinge at one end, and is supported by a cord, 13 feet long, attached to the other end and to a fixed point situated 20 feet vertically above the hinge. Find the tension of the cord, assuming the weight of the beam to be 140 lbs.

3. A uniform beam AB , 20 feet long, is supported by props at C and D , two points at distances 4 feet from A and 6 feet from B , respectively. If a load of 1 ton be placed at each extremity of the beam, calculate the magnitude of the moment which tends to break the beam at the middle point of CD : assuming the weight of the beam to be 2 tons.

4. A ladder is placed with one end on a rough horizontal plane and the other against a rough vertical wall; find, by geometrical construction, or otherwise, the limiting position of equilibrium, being given the coefficients of friction and the centre of gravity of the ladder.

If an additional load be placed at any point on the ladder, in this limiting position, find whether the equilibrium will be disturbed or not.

5. Three forces acting on a rigid system are in equilibrium, prove that their directions lie in the same plane, and either pass through a common point or are parallel.

A uniform cubical block is sustained on a rough inclined plane by a rope, which is parallel to the plane and is attached to the middle point of the upper edge of the cube, which is horizontal. The rope lies in the vertical plane, which contains the centre of the cube and is perpendicular to the inclined plane. Determine the greatest inclination of the plane consistent with equilibrium.

6. Two particles start simultaneously from different points, in given directions, with uniform velocities. Show, by geometrical construction, how to find their relative distance at the end of any time; and determine when this distance is least.

7. Prove that the path of a projectile (*in vacuo*) is a parabola, and that the velocity at any point is equal to that due, under the action of gravity, to the vertical distance of the point from the directrix.

A train is moving with a velocity of 30 miles an hour when a ball is dropped from the roof of one of the carriages 10 feet above the earth. Show how to find the focus and directrix of the parabolic path described by the ball, relatively to the earth.

8. Enunciate accurately Newton's Laws of Motion.

A train is moving on a horizontal railroad. Assuming the weight of the train (exclusive of the engine) to be 120 tons, and the resistance arising from friction, etc., to be 10 lbs. per ton, find the tension of the couplings of the carriage which is attached to the engine, (1) when the velocity of the train is uniform, (2) when it is moving with an acceleration of 4 feet per second, per second.

9. Two imperfectly elastic spherical bodies, whose centres are moving in the same straight line with given velocities, impinge on each other: show how to find their velocities immediately after the impact.

Two spheres meet directly with equal and opposite velocities, find the ratio of their masses in order that one of them should be brought to rest by the collision, (1) when perfectly elastic, (2) for coefficient of resilience e .

10. A ball moving with a velocity of 500 feet per second has its velocity reduced by 50 feet after penetrating 1 inch into a plank. Find how far it will penetrate into the plank before being stopped, assuming the resistance of the plank to be uniform.

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

JUNE, 1894.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

Ordinary abbreviations may be employed; but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.]

1. The opposite sides and angles of a parallelogram are equal to one another.

2. The complements of the parallelograms which are about the diameter of any parallelogram, are equal to one another.

A point E is taken in the side AB of the parallelogram $ABCD$ and ED and EC are joined; prove that, if the line HK parallel to AB cuts ED and EC in F and G respectively, the parallelogram $AHKB$ will be double of either of the triangles EDG or ECF .

3. If a straight line be divided into two equal parts and also into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line.

4. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

The line AB is bisected in C and produced to D , so that the square on CD is equal to the sum of the squares upon AB and BC , prove that the rectangle AD , DB is equal to the square on AB .

Express the ratio of BD to AB algebraically.

5. State, *without proof*, the difference between the square on one side of a triangle and the sum of the squares of the two remaining sides.

A point P is taken on the circumference of the circle APB , whose centre is O , and with P as centre, another circle, QAB , is described, cutting the former in A and B . If QM be drawn from Q , any point on QAB outside of APB , perpendicular to AB produced, prove that twice the rectangle of QM and OP is equal to the difference of the squares on OQ and OA .

6. The angle at the centre of a circle is double of the angle at the circumference on the same base, that is, on the same arc.

7. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

8. Describe a circle about a given triangle.

The straight line AB of given length moves so that its extremities are respectively upon the two fixed straight lines OC and OD meeting at O . Prove that the centre of the circle circumscribing the triangle OAB lies upon the circumference of a circle whose centre is O .

9. Inscribe an equilateral and equiangular pentagon in a given circle.

10. If the vertical angle of a triangle be bisected by a straight line which also cuts the base, the segments of the base shall have the same ratio which the other sides of the triangle have to one another, and if the segments of the base have the same ratio which the other sides of the triangle have to one another, the straight line drawn from the vertex to the point of section shall bisect the vertical angle.

11. If four straight lines be proportionals, the rectangle contained by the extremes is equal to the rectangle contained by the means; and if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines are proportionals.

12. The diameter BCA of the circle $APQB$ whose centre is C , is produced through A to O , and from O the line OPQ is drawn, cutting the circle in P and Q , prove that the triangles OPA and OQB are similar, and prove that if the circle circumscribing the triangle PCQ meets OC in D then

- (1) The point D will be fixed for all directions of the line OPQ ;
- (2) The ratios $OA : AD$ and $OC : CP$ will be equal.

II. ARITHMETIC.

[N.B.—*The working as well as the answers must be shown.*]

1. Find the value of

$$\frac{5\frac{1}{2} - 3\frac{5}{8}}{2\frac{1}{2} + 3\frac{3}{4}} \times 79\frac{1}{2}.$$

2. Divide '0468 by 29.25.

3. What is the interest on £775 for $3\frac{1}{2}$ years at $2\frac{3}{4}$ per cent. per annum?

4. Find the value of £1.69375.

5. What fraction of an acre is 28 poles?

6. Find by Practice the value of 13 lbs. 7 ozs. 5 dwts. 8 grs. of silver at 3s. 9d. per oz.

7. Find the least common multiple of 132, 165, 220.

8. A cube of metal, each edge of which measures $\frac{5}{8}$ of an inch, weighs '625 lb. What is the length of each edge of a cube of the same metal which weighs 40 lbs.?

9. If 9 men do $\frac{3}{5}$ of a piece of work in 14 days, working 10 hours a day, how many extra men must be employed to finish the work in 5 days more if all of them are to work only 8 hours a day?

10. Prove that the difference of the squares of any two odd numbers is divisible by the double of their sum, and that, if the numbers are consecutive odd numbers, the difference of their squares is a multiple of 8. (N.B.—*You may take any two odd numbers which you like to select, show that the propositions are true for those numbers, and then extend your reasoning to all odd numbers.*)

11. A's money is to B's money as 3 : 7 ; and if B pays A £345, the proportion will be 3 : 5. How much money has each?

12. If 9 men and 6 boys can do in 2 days what 5 men and 7 boys could do in 3 days, in what time could 2 men and 5 boys do the same?

13. A dealer has two sorts of tea, one of which he could sell at 1*s.* 8*d.* per lb. and make 25 per cent. on his outlay, the other at 2*s.* 3*d.* and make 12½ per cent. What profit per cent. will he make if he mixes them in equal quantities and sells the mixture at 1*s.* 11*d.* per lb. ?

14. Goods are imported from abroad at an expense equal to 35 per cent. of the cost of production ; and the importer makes 15 per cent. on his whole outlay by selling them to a tradesman at £7. 15*s.* 3*d.* per ton. Find the cost per ton of production.

III. ALGEBRA.

(Up to and including the Binomial Theorem, the theory and use of Logarithms.)

[N.B.—Great importance will be attached to accuracy.]

1. Divide $x^5 + x^4 + 4x^3 + 21x^2 + 23x - 40$ by $x^2 + 4x + 5$.

2. Resolve into factors the expressions :

$$x^3 - 8y^3, \quad x^2 - 24xy + 128y^2, \quad \text{and} \quad x^4 + x^2y^2 + y^4.$$

3. Find the Highest Common Factor of the expressions :

$$x^4 - 8x^3 + 13x^2 - 30x + 8 \quad \text{and} \quad x^4 - 4x^3 - 11x^2 - 50x + 16.$$

4. Prove that $(b-c)^5 + (c-a)^5 + (a-b)^5$

is divisible by $(b-c)(c-a)(a-b)$,

and find the other factor.

5. Find the square roots of the expressions

$$(i.) \quad n(n+1)(n+2)(n+3) + 1.$$

$$(ii.) \quad 24 + \sqrt{572}.$$

6. Solve the equations

$$(i.) \quad \frac{x+a}{x-c} + \frac{x+c}{x-a} = 2;$$

$$(ii.) \quad x^2 + 3x + 4\sqrt{x^2 + 3x - 3} = 48.$$

7. If α and β are the roots of the equation $x^2 + px + q = 0$, prove that $\alpha + \beta = -p$ and $\alpha\beta = q$, and find, in terms of p and q , the equation of which the roots are

$$\alpha + 2\beta \quad \text{and} \quad \beta + 2\alpha.$$

8. At present the ratio of B's age to A's age is the ratio 5 : 2, but in 30 years' time the ratio will be 35 : 23; find their ages.

9. Find the least possible positive value of the expression

$$4x + \frac{9}{x},$$

and the greatest possible value of the expression

$$\frac{x+2}{4x^2+16x+25}.$$

10. If the expression $x(1-5x+6x^2)^{-1}$ be expanded in powers of x , prove that the coefficient of x^r is $3^r - 2^r$.

11. Write down an expression for the number of combinations of n things taken r together, and, if ${}_nC_r$ represent this expression, prove, by general reasoning or otherwise, that

$${}_nC_r = {}_nC_{n-r}, \quad \text{and} \quad {}_{n+1}C_r = {}_nC_r + {}_nC_{r-1}.$$

12. Having given

$\log_{10} 5 = .6989700$, $\log_{10} 7 = .8450980$, and $\log_{10} 11 = 1.0413927$, find the logarithms, to the base 10, of 385 and $\frac{175}{121}$, and solve approximately, to two places of decimals, the equation

$$5^x \cdot 7^{x-1} = 11^{x+2}.$$

IV. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy.]

$$[\pi = \frac{22}{7}.]$$

1. Explain what is meant by the *circular measure* of an angle, and find the circular measure of an angle of x degrees.

A wire AB , 1 foot in length, is bent so as to form an arc of a circle whose diameter is four inches; find the angle subtended at the centre of the circle by the chord AB .

2. Express the Trigonometrical ratios of an angle in terms of the secant, the angle being less than a right angle.

If in a triangle ABC ,

$$CA = CB = 2, \text{ and } AB = 3;$$

find the value of

$$(\sin A - \cos A)(\sec A + \operatorname{cosec} A).$$

3. Determine the values of the Trigonometrical ratios for an angle of 60° , and for an angle of 30° .

A ladder rests against a vertical wall at an angle of 60° with the horizon, and when the foot is drawn back 18 feet further from the wall the inclination to the horizon is found to be 30° . Find the length of the ladder.

4. Prove that $\cos(90^\circ + A) = -\sin A$ for the case when A is less than two right angles.

Write down the values of $\sin 225^\circ$, $\cos 210^\circ$, $\tan 315^\circ$, and $\operatorname{cosec} 420^\circ$.

5 Show geometrically that $\cos(A + B) = \cos A \cos B - \sin A \sin B$, where A, B are two positive angles whose sum is less than a right angle.

Find the value of $\cos 75^\circ$.

6. Express $\sin 3A$ and $\cos 3A$ in terms of $\sin A$ and $\cos A$, and show that

$$\cos 3A + \sin 3A = (\cos A - \sin A)(1 + 2 \sin 2A).$$

7. Prove the following identities :

$$(i.) (x \tan a + y \cot a)(x \cot a + y \tan a) = (x + y)^2 + 4xy \cot^2 2a.$$

$$(ii.) \cos \beta \cos(2a - \beta) = \cos^2 a - \sin^2(a - \beta).$$

$$(iii.) \cos a + \cos 3a + \cos 5a + \cos 7a = \frac{1}{2} \sin 8a \operatorname{cosec} a.$$

8. In any triangle ABC find $\tan \frac{1}{2}A$ in terms of the sides a, b, c .

Find the angles of the triangle whose sides are proportional to 3, 5, 7.

9. Show how to solve a triangle when two sides and (i.) an angle opposite to one of the sides, and (ii.) the included angle, are given.

From a boat at sea the angle subtended by the line joining two fixed objects A, B on land is observed to be α ; after sailing x yards directly towards A the angle subtended by AB is found to be β , and then after sailing y yards directly towards B , the angle is found to be γ . Find an expression for the distance AB .

10. In a triangle ABC , $a = 35$, $b = 43$, and $C = 75^\circ 10' 40''$, find the angles A and B .

11. Find the area enclosed by 200 hurdles placed so as to form a regular polygon of 200 sides, the length of each hurdle being 6 feet.

12. A leaden sphere one inch in diameter is beaten out into a circular sheet of uniform thickness = $\frac{1}{100}$ th inch. Find the radius of the sheet.

V. STATICS AND DYNAMICS.

[You may assume that $g = 32$ feet per second per second.]

1. State accurately the principle known as that of the *Triangle of Forces*.

A particle is acted upon by three forces of given magnitudes; show how these forces must be arranged so as if possible to produce equilibrium, when

- (i.) the forces have magnitudes represented by the numbers, 6, 11, 18.
- (ii.) their magnitudes are represented by 8, 15, 17.

2. Let O be the position of a particle, and OA a right line drawn through O . Find the magnitude and direction of the resultant of forces proportional to

10, 18, 20, 16,

acting on the particle, when their lines of action make with OA angles of

$0^\circ, 30^\circ, 90^\circ, 135^\circ,$

respectively, all measured in the same sense (*i.e.*, all outwards from O , or all inwards towards O).

3. A body whose mass is 520 pounds is placed on a smooth inclined plane, the tangent of whose inclination to the horizon is $\frac{5}{12}$; find the force necessary to sustain the body

- (i.) when this force is horizontal;
- (ii.) when it acts along the inclined plane.

4. Define a couple, and show that its moment is the same about all points in its plane.

Prove that the sum of the moments of any two forces in the same plane about any point in the plane is equal to the moment of their resultant about the point.

5. Two parallel forces, P , Q , act on a rigid body; find the magnitude and line of action of their resultant (1) when they act in the same direction; (2) when they act in opposite directions.

Two parallel forces of 20 and 25 pounds' weight, of opposite senses, act on a rigid body, the perpendicular distance between their lines of action being 4 inches; find the resultant.

6. Show how to find the position of the centre of gravity of a given system of particles whose masses are $m_1, m_2, m_3, \dots$ occupying given positions.

At each vertex of a triangle is placed a particle whose mass is proportional to the length of the opposite side ; show that the centre of gravity is the centre of the inscribed circle.

From a thin uniform circular plate of radius 13 inches is cut out a circular plate of radius 5 inches, the centre of the latter being 4 inches distant from that of the former ; find the position of the centre of gravity of the remainder.

7. Describe the screw press, and find the condition for the equilibrium of the effort and resistance ("power" and "weight") applied to it when there is no friction.

8. Define *acceleration*. State clearly what is meant by saying that g is about 32 feet per second per second, and describe any method by which this value of g has been obtained.

Which is the greater acceleration, 15 miles per hour per minute, or $\frac{1}{8}$ feet per second per second ?

9. If a particle moves in a right line with constant acceleration, a , having had an initial velocity, u , show that the distance travelled in time t is given by the equation

$$s = ut + \frac{1}{2}at^2.$$

A bullet is fired vertically upwards with a velocity of 496 feet per second ; 3 seconds afterwards another is fired vertically upwards from the same point with a velocity of 568 feet per second ; when and where will they meet ?

10. Define an *absolute unit of force*, and, in particular, the *dyne* and the *poundal*.

What force uniformly applied to a mass of 12 pounds will give it an acceleration of 8 feet per second per second ? (Express the force both in pounds' weight and in poundals.)

11. From a given point on a horizontal plane is projected a particle with a velocity u at an elevation α ; find the range on the plane.

If the particle is projected with a velocity of 500 feet per second at an angle of elevation whose tangent is $\frac{4}{3}$, find the range. Find also the magnitude and direction of the velocity 10 seconds after the time of projection.

FURTHER EXAMINATION.

VI. PURE MATHEMATICS.

[Full marks may be obtained for about two-thirds of this paper.]

1. Sum up the conditions under which Euclid says that two triangles are equal.

ABC is a triangle and D a point in BC such that AD bisects the angle A . If O be the centre of a circle which touches AB at A and also passes through D , prove that OD and AC are at right angles and find the magnitude of the angle AOD .

2. Apply the theory of *geometrical progression* to the evaluation of a mixed recurring decimal.

Show that the sum of $\frac{1}{37}$ and the series

$$\frac{38 \times 18}{37^2} + \frac{38 \times 18^2}{37^3} + \frac{38 \times 18^3}{37^4} + \dots \text{ad inf.}$$

is unity.

Employ logarithms to evaluate the tenth and twentieth terms of the series.

3. A clock strikes at intervals of one second. Determine the intervals as they appear to men travelling in express trains at 60 miles an hour directly towards and directly away from the clock respectively. *The velocity of sound may be taken as 1100 feet per second.*

4. Find the greatest coefficient in the expansion of

$$(x+y)^n.$$

Prove that the greatest coefficients in the expansion of the trinomial

$$(x+y+z)^{3n+1}$$

have the value

$$\frac{(3n+1)!}{(n!)^3(n+1)}.$$

N.B.— $n!$ represents the product $n(n-1)(n-2) \dots 3 \cdot 2 \cdot 1$.

5. In any triangle show that, in the usual notation,

$$\cos A + \cos B = \frac{a+b}{c} (1 - \cos C),$$

$$\cos B - \cos A = \frac{a-b}{c} (1 + \cos C).$$

6. Find the volume of a pyramid on a triangular base and deduce that the volume of a sphere is the product of the area of its surface and one-third of its radius.

A sphere is cut by a horizontal plane. If P be a point in the perimeter of the section and A the highest point of the sphere, show that the surface of the sphere above the cutting plane has an area equal to that of a circle of radius AP .

7. In the triangle ABC , sides 4, 3, 5 AB produced is a tangent, BC the axis and AC the tangent at the vertex of a parabola.

Draw the triangle in your book and construct geometrically for (i.) the focus, (ii.) the directrix, (iii.) the point P on the parabola at which AB is tangent, (iv.) the other extremity of the focal chord through P , (v.) the extremities of the latus rectum.

8. In an ellipse QQ' is the chord of contact of tangents from an external point P ; CK is the perpendicular on QQ' from the centre C ; and PG is the perpendicular to QQ' , through P meeting the major axis in G . Prove that the semi-minor axis is a mean proportional to KC and PG .

9. Write down the usual forms of equation to a straight line.

Through a given point P whose coordinates are (p, q) a straight line is drawn intersecting the axes in A and B so that P is the middle point of AB . Determine the equation of AB .

10. Find the general equation of the circle when the coordinate axes are inclined to one another at an angle ω .

The axes being rectangular investigate the condition that must be satisfied by the parameters of the circle in order that it may be possible to find a point or points on the circle at equal perpendicular distances from the axes.

11. In the parabola

$$y^2 = 4ax$$

find the equation to the circle passing through the vertex and the extremities of the latus rectum.

Find also the coordinates of the points where the circle is intersected by those normals to the parabola which make an angle of 45° with the axis.

12. C is the centre of an ellipse and BB' the minor axis. If S be the focus which (to origin at the centre and coordinate axes coincident with the axes of the ellipse) has a positive abscissa; and $B'S$ be produced to meet the curve in P ; show that CP makes an angle ϕ with the major axis such that

$$2e \tan \phi = (1 - e^2)^{\frac{3}{2}}.$$

13. Find the equation to the tangent at any point of the hyperbola

$$4xy = a^2 + b^2.$$

If two tangents at right angles intersect at P find the locus of P .

VII. MECHANICS.

[Full marks may be obtained for about two-thirds of this paper. Great importance is attached to accuracy. N.B.— g may be taken = 32.]

1. If three forces acting on a particle are in equilibrium, each force is proportional to the sine of the angle between the other two.

$ABCD$ is a quadrilateral, having the angles at A and D right angles, and $CB = CD$. Forces P , Q , and R , acting along AB , CA , and AD respectively, are in equilibrium. If

$$P = \frac{2}{\sqrt{3}}R,$$

show that $AB = \frac{1}{2}BC$ or $\frac{3}{2}BC$.

2. Prove that the algebraical sum of the moments of a number of coplanar forces acting on a particle about any point in their plane is equal to the moment of their resultant about that point.

P is the orthocentre of a triangle ABC . Forces act along AP , BP , CP , and are proportional to $\sin(A + \theta)$, $\sin(B + \theta)$, $\sin(C + \theta)$. Prove that their resultant passes through the centre of the circumscribed circle. What is the value of θ when the forces are in equilibrium?

3. Two equal uniform rods, AB , BC , each of weight W , are hinged together by a smooth hinge at B , and rest on a smooth cylinder of radius b ,

whose axis is horizontal. The rods touch the cylinder at points distant one-third of their length from B . Find the length of the rods, and prove that the bending moment at the point of contact of either rod with the cylinder is $\frac{2}{3\sqrt{3}} Wb$.

4. A heavy circular wheel is suspended in a vertical plane from a point B in a rough vertical wall, by a smooth weightless wire through its centre O , and rests at right angles to the wall and touching it at the point A . A string is fixed to the rim of the wheel and passes over it on the side away from A . If the inclination (α) of the wire OB to the horizon is less than 2β , where $\tan \beta$ is the coefficient of friction between the wheel and the wall, show that a force, however great, applied along the string will not turn the wheel unless the direction of the string makes with BO produced an angle less than

$$\sin^{-1} \frac{\sin(\alpha - \beta)}{\sin \beta}.$$

(Positive angles are measured from OA to OB .)

5. Find the position of the centre of gravity of a cylindrical bar with circular ends, whose density at any point is proportional to the square of the distance of the point from the end of the bar.

A cone of vertical angle 2α rests with its base on a rough plane inclined at an angle β to the horizon, the coefficient of friction (μ) being greater than $\tan \beta$. A gradually increasing force is applied at the vertex of the cone parallel to the plane and downwards. Show that, when the force is large enough to disturb equilibrium, the cone will tilt over or slide down according as $\tan \beta$ is less or greater than $\frac{4}{3}(\mu - \tan \alpha)$.

6. A particle initially at rest is acted on by a force constant in direction and magnitude. Prove that the kinetic energy of the particle at any time is equal to the work done on it by the force.

A particle moves from rest down a rough plane inclined at an angle 2θ to the horizon, $\tan \theta$ being the coefficient of friction. Prove that in moving over a length s of the plane it acquires the same velocity as in falling freely through a distance $s \tan \theta$.

7. An engine draws a train whose weight (exclusive of the engine) is 100 tons. The power of the engine is such that when running on the level it exerts a pull of 2 tons weight on the front carriage, and the resistance due to friction, etc., is 11.2 lbs. per ton. Show that if the engine draws the same train from rest up an incline of 1 in 300 it will in one minute acquire a velocity slightly exceeding $15\frac{3}{4}$ miles per hour.

8. A bullet is fired with a velocity of 800 feet per second. Find (a) its greatest possible range on the horizontal plane through the point of projection; and (b) the height (approximately) to which the bullet ascends when the range is one-tenth of the greatest possible range.

9. A mass of weight W rests on the smooth surface of a horizontal table, also of weight W , and is connected by a weightless string, passing over a smooth pulley at the edge of the table, with a weight $2W$ hanging freely, which is allowed to fall, the string being initially taut. If the table does not move, find the tension of the string, and show that the coefficient of friction between the table and the floor is not less than $\frac{1}{4}$.

10. Two equal imperfectly elastic balls moving in the same straight line impinge directly upon one another. Find the change of kinetic energy produced by the impact.

A billiard ball A moving parallel to one side of the table, strikes another ball B (initially at rest) in the centre. After B has struck the cushion and then struck A again, the velocity of A is three-fourths of its initial velocity, but in the opposite direction. Show that, if e is the coefficient of elasticity between the balls and also between a ball and the cushion,

$$e^3 + e^2 + 3e = 4.$$

MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

Royal Military Academy, Woolwich,

NOVEMBER, 1894.

OBLIGATORY EXAMINATION.

I. EUCLID (BOOKS I.—IV. AND VI.).

[Ordinary abbreviations may be employed, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.]

1. Define *plane rectilineal angle*, *circle*, *gnomon*, *similar rectilineal figures*.

2. If one side of a triangle be produced, the exterior angle is greater than either of the interior opposite angles.

3. The straight lines which join the extremities of two equal and parallel straight lines towards the same parts are also themselves equal and parallel.

$ABCD$ is a quadrilateral ; show that, if the four parallelograms $BCDP$, $CDAQ$, $DABR$, $ABCS$ be completed, the four straight lines AP , BQ , CR , DS will be equal and parallel.

4. If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the two parts.

WOOLWICH ENTRANCE EXAMINATION.

Show also that if a straight line be divided into any three parts, the square on the whole line is equal to the squares on the three parts, together with twice the three rectangles whose sides are the three parts taken two and two together.

5. Describe a square equal to a given rectilineal figure.

6. If a straight line drawn through the centre of a circle bisect a straight line in it which does not pass through the centre, it cuts it at right angles; and if it cut it at right angles, it bisects it.

7. Straight lines in a circle which are equally distant from the centre are equal to one another.

8. If a straight line touch a circle, and from the point of contact a straight line be drawn cutting the circle, the angles which this line makes with the line touching the circle are equal to the angles which are in the alternate segments of the circle.

Four circular coins, of different sizes, are placed upon a table so that each one touches two, and only two, of the remaining three; show that the four points of contact lie on a circle.

9. Inscribe a circle in a given triangle.

Prove that the centre of this circle lies inside each of the three circles described on the three sides of the triangle as diameters.

10. Describe a circle about a given equilateral and equiangular pentagon.

11. Triangles which have one angle of the one equal to one angle of the other and the sides about the equal angles reciprocally proportional, are equal to one another.

Two straight lines AOC , BOD intersect in O and the lines AB , CD are drawn. From the greater of the two triangles AOB , COD cut off a part equal to the less by a straight line drawn through the point O .

12. Parallelograms about the diameter of any parallelogram are similar to the whole parallelogram and to one another.

II. ARITHMETIC.

[N.B.—*The working as well as the answers must be shown.*]

1. Simplify $\frac{2\frac{5}{9} \text{ of } 1\frac{17}{48} - 3}{4\frac{2}{11} - 2\frac{3}{4}} \div 2\frac{2}{21}$.
2. Find the value of $(2.37 \times .093) \div .0005$.
3. A sum of £237. 18s. 4d., lent at simple interest, amounts in three years to £270. 0s. 8½d. What is the rate per cent. ?
4. Find the value in cwts. qrs. and lbs. of .0234375 of 9½ tons.
5. What fraction is $28\frac{1}{3}$ cubic inches of a cubic foot ?
6. Find the value of 13 acres 2 roods 17½ perches at £14. 14s. 8d. per acre.
7. Find the greatest common measure of 10058, 4982, and 9823.
8. Justify, from first principles, each step of the process of addition of vulgar fractions and deduce the rule for the addition of decimals.
9. What was the cost of goods on which a man lost 20 per cent. by selling them for £64 ?
10. A clock set right at noon on Tuesday loses at the rate of 192 seconds in 10 hours. What is the true time on the following Friday afternoon when the reading of this clock is 2 hours 36 minutes ?
11. Determine, without performing the divisions, the remainders that result from dividing 48909661 by 8, 16, 25, 9, and 11 ; and give a brief explanation of the *reason* from which you draw your conclusion in the first three cases.
12. If it costs the same amount to keep 4 horses or 9 oxen, and 5 horses can do as much work as 8 oxen, which will it be more profitable to employ—20 horses and 32 oxen, or 12 horses and 48 oxen ?

WOOLWICH ENTRANCE EXAMINATION.

13. The incomes of two men would be equal if one were increased 7 per cent. and the other diminished $7\frac{1}{2}$ per cent., and the sum of their incomes is £418. 19s. What is the income of each?

14. Three trains start from a town *A* at 12.0, 12.5, and 12.10, and travelling each at a uniform rate by 3 different routes of the same length to a town *B*, are observed to pass a signal box at *B* exactly abreast of each other at 12.50. If at 12.20 the sum of the distances traversed by the three trains is 36 miles 7 furlongs, how far from *A* is the signal box at *B*?

III. ALGEBRA.

(Up to and including the Binomial Theorem; the theory and use of Logarithms.)

[N.B.—*Great importance will be attached to accuracy.*]

1. Divide $x^6 - a^6$ by $x^2 - ax + a^2$.

Show that $x^6 - a^6$ is divisible by $x^2 + px + \frac{1}{3}p^2$, if $p^6 + 27a^6 = 0$.

2. Express

$$\frac{\frac{1}{x} + \frac{1}{y-z}}{\frac{1}{x} - \frac{1}{y-z}} \left\{ 1 - \frac{y^2 + z^2 - x^2}{2yz} \right\}$$

in its simplest form.

3. Prove, by the method of finding the Greatest Common Measure, that $2x^2 - x - 6$ is a factor of

$$2x^5 + 3x^4 - 32x - 48,$$

and

$$2x^6 - x^5 - 6x^4 + 2x^3 - 3x^2 - 5x + 6.$$

4. Show, by any method, that

$$a^3(b-c) + b^3(c-a) + c^3(a-b)$$

contains $b-c$, $c-a$, $a-b$ as factors, and find the remaining factor.

5. Find the values of x , y , z from the equations

$$x + 2y - z + 4 = 0,$$

$$3x + 4y + z - 1 = 0,$$

$$5x + 6y - 3z + 18 = 0.$$

6. Simplify the equation

$$\frac{2x-1}{3} - \frac{4x^2-1}{x+3} = \frac{x-3}{x+3} \cdot \frac{10x+1}{2x+3},$$

$$x+1 - \frac{5x^2-9}{3(x-3)}$$

and solve it.

7. Find the condition that the roots of the equation

$$ax^2 + bx + c = 0$$

shall be equal.

Determine the values of k for which the equation

$$12(k+2)x^2 - 12(2k-1)x - 38k - 11 = 0$$

will have equal roots.

8. Prove that, if a and l are the first and last terms of an arithmetical progression containing n terms, the sum of the series is

$$\frac{1}{2}n(a+l).$$

The sum of 5 terms of an arithmetic series is 10, and the sum of 17 terms is -17; find the series.

9. Expand

$$\frac{1}{\sqrt[3]{1-x}}$$

in a series ascending by powers of x , as far as x^3 , by the Binomial Theorem, and write down an expression for the n^{th} term of the series.

10. A and B are two stations on a railway, 90 miles apart. At the same instant one train passes through A towards B and another through B towards A , with different but constant speeds. They pass each other at C , and AC is 10 miles longer than BC ; also, the first reaches B half-an-hour before the second reaches A ; find their speeds.

11. Define the logarithm of any number to a given base, a .

Find, from a table of common logarithms, the logarithm of 125 to the base $4\frac{1}{2}$. Give, without proof, to four decimal places, the value of the modulus which converts logarithms to base 10 into logarithms to the Napierian base.

12. If e is the Napierian base, prove, by any method, that

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

Hence show that, for any number, n ,

$$\log_e n = 2 \left\{ \frac{n-1}{n+1} + \frac{1}{3} \left(\frac{n-1}{n+1} \right)^3 + \frac{1}{5} \left(\frac{n-1}{n+1} \right)^5 + \dots \right\}.$$

IV. PLANE TRIGONOMETRY AND MENSURATION.

(Including the Solution of Triangles.)

[N.B.—Great importance will be attached to accuracy. $\pi = \frac{22}{7}$.]

1. Prove that the angle subtended at the centre of a circle by an arc equal to the radius of the circle is of the same magnitude for all circles.

Find this angle in degrees.

2. Write down formulæ for $\sin(A+B)$, $\cos(A-B)$, and $\tan(A+B)$, in terms of the trigonometrical functions of A and B .

Find $\cos 75^\circ$ and $\sin 18^\circ$.

3. Find a value of A , positive and less than 90° , satisfying the equation
 $(\sin A^\circ - \cos A^\circ)(\sec A^\circ - \operatorname{cosec} A^\circ) = 2$.

4. Prove that for all values of A and B

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}.$$

If the sum of two angles be always equal to α , where α is positive and not greater than 180° , prove that the sum of their cosines will be never greater than $2 \cos \frac{\alpha}{2}$ and never less than $2 \cos^2 \frac{\alpha}{2}$.

5. Prove that for all values of A and B

$$\cos B - \cos A = 2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}.$$

Find the numerically smallest value of θ , different from zero, satisfying the equation

$$\cos 3\theta - \cos 4\theta = \cos 5\theta - \cos 6\theta.$$

6. Prove that $\sin 33^\circ + \cos 63^\circ = \cos 3^\circ$;
 also that, if $\cot A \cot B = 2$,
 then $\cos(A-B) = 3 \cos(A+B)$.

7. In the triangle ABC prove that

$$a \cos B + b \cos A = c.$$

$$a \sin B - b \sin A = 0.$$

$$a^2 + b^2 - 2ab \cos C = c^2.$$

8. Given $A = 42^\circ$, $a = 141$, $b = 172.5$,
find all solutions of the triangle ABC .

9. From the point O the three straight lines OA , OB , OC are drawn in the same plane, of lengths 1, 2, 3 respectively, and with the angles AOB and BOC each equal to 60° . Find the angle ABC correct to one minute.

10. Find the area of the greatest circle which can be cut out of a triangular piece of paper whose sides are 3, 4, 5 feet respectively.

11. A conical extinguisher, whose section through the vertex is an isosceles triangle with vertical angle 30° , is placed over a cylindrical candle whose diameter is one inch, and rests so that the point of contact of the top of the candle with each generating line of the cone bisects that line. Find the whole inside surface of the extinguisher.
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V. STATICS AND DYNAMICS.

[It may be assumed that $\pi = \frac{22}{7}$, and that $g = 32$, when a foot and a second are the units of length and time.]

1. Explain why it is that forces can be completely represented by straight lines.

$ABCD$ is a square; find the resultant of the forces represented by the straight lines AB , AC , and AD .

2. Enunciate and prove the theorems of the triangle of forces and the polygon of forces, and state whether the converses of these theorems are true.

3. A heavy pole, weighing 140 lbs., is carried on the shoulders of two men, one at each end; the centre of gravity of the pole being two feet from one end and five feet from the other, find the weight supported by each man.

Also find what would be the effect of placing each man one foot nearer to the centre of gravity of the pole.

4. Find the ratio of the power to the weight when there is equilibrium in a system of three moveable pulleys, each of which is supported by a separate string, and in which the free portions of the strings are vertical.

Also, if the weights of the pulleys, supposed to be equal, are taken into account, find the relation between the power, the weight, and the weight of a pulley.

5. A heavy uniform rod is supported by a string fastened to its ends, of double its own length, which passes over a smooth horizontal rail. Find the tension of the string first, when the rod is hanging at rest in a vertical position, and secondly, when the rod is at rest in a horizontal position.

6. Explain what is meant by saying that a point is moving in a straight line with uniform acceleration, and show how this acceleration is measured.

What is the measure of the acceleration of a body falling freely when eight feet and half a second are the units of length and time?

7. A body is projected vertically upwards with the velocity of 256 feet per second ; find the greatest height to which it rises and the time in which it will return to the point of projection.

Also find the times, during the ascent and descent, at which it passes the level of 768 feet above the point of projection.

8. Prove that the path of a projectile is a parabola, and that, if u is the horizontal component of the velocity of projection, the latus rectum of the parabola is equal to $\frac{u^2}{16}$.

9. The top of the spire of a church, standing on a level plane is 200 feet above the plane. From a position on the plane, at the distance of 400 feet from the vertical line through the top of the spire, a bullet is fired off so as to pass horizontally just over the top of the spire. Find the initial direction and the initial velocity of the bullet.

10. Find the direction and magnitude of the acceleration of a point moving uniformly in a circle.

A mass of 7 pounds, on a smooth horizontal plane, is fastened to one end of a string, 7 feet in length, and the other end is fastened to a fixed peg on the plane. The string is then straightened, and the particle is projected horizontally, at right angles to the string, with such a velocity as to describe its circular path in $5\frac{1}{2}$ seconds. Find the tension of the string in poundals, and also in pounds' weight.

FURTHER EXAMINATION.

VI. PURE MATHEMATICS.

[Full marks may be obtained for about two-thirds of this paper.]

1. Show that the two perpendiculars, erected at the extremities of any chord of a circle, meet any diameter of the circle at two points equidistant from the centre; and contain a rectangle equal to the difference of the squares on the radius and on half the interval they intercept on the diameter.

2. Describe a circle passing through two given points and intercepting, on a given line, a segment of given length.

3. Define a homogeneous integral function of any number of variables.

Write down the most general function, of degree 4, in 3 variables.

4. If the increase in population be 8 per cent. every decade, the rate of increase being constant, find the population of a town of 100,000 inhabitants 5 years hence. Find also, employing a table of logarithms, the percentage of increase per annum.

5. Find the coefficient of xyz in the expansion of

$$\{(1-x-y-z)(1-x-y-z+4xyz)\}^{-\frac{1}{2}}$$

as a rational integral function of x , y , and z .

6. If the circle escribed to the side BC of a triangle ABC touch AB and AC produced in D and E respectively, prove that $AD = AE =$ half the sum of the sides of the triangle.

The longest side of a triangular plot of ground is 100 yards, the perimeter is 250 yards and one angle is 40° . Determine the remaining angles.

$$\begin{aligned} 7. \quad \text{If} \quad r &= p(1 + e \cos \phi)^{-1} \\ r' &= p(1 + e \cos \phi')^{-1} \\ r'' &= p(1 + e \cos \phi'')^{-1} \end{aligned}$$

express

$$\frac{\sin(\phi'' - \phi')}{r} + \frac{\sin(\phi - \phi'')}{r'} + \frac{\sin(\phi' - \phi)}{r''}$$

in a form adapted to logarithmic computation and evaluate it where

$$\phi = 17^\circ 4', \quad \phi' = 22^\circ 27', \quad \phi'' = 38^\circ 19', \quad p = 21.$$

8. There is a rectangular plot of ground. Show how, by means of a cord, an ellipse may be inscribed so as to touch the sides at the middle points.

Prove the propositions on which the construction depends.

9. Give a geometrical construction for drawing tangents to an hyperbola from an external point.

10. Find the equation to a straight line passing through a fixed point (h, k) and making an angle of $\pi/3$ with the axis of x .

If the straight line rotate, in a counter clock-wise direction, about the fixed point through an angle of $1'$, show that the intercept on the axis of y is diminished by a length equal to $\frac{\pi}{2700}h$ approximately.

11. If the point (h, k) do not lie on the perimeter of the circle

$$(x - \alpha)^2 + (y - \beta)^2 - \rho^2 = 0$$

interpret the expression

$$(h - \alpha)^2 + (k - \beta)^2 - \rho^2$$

geometrically.

Transpose the above equation to polar coordinates and find the angle between the two tangents from the pole.

12. Find an equation to a parabola.

Prove, analytically, that, at every point of the curve, the diameter and focal radius make equal angles with the tangent.

13. Given the axes of an ellipse, find expressions for

- the distance between the foci.
- the distance between the directrices.
- the latus rectum.
- the product of the lengths of the perpendiculars from the foci on any tangent.

14. Explain carefully the nature of an asymptote to an hyperbola.

Find the equation to a curve, of this description, such that the smaller angle between the asymptotes is 45° , and the distance between the foci is 10 units of length.

14. Explain carefully the nature of an asymptote to an hyperbola.

Find the equation to a curve, of this description, such that the smaller angle between the asymptotes is 45° , and the distance between the foci 10 units of length.

VII. MECHANICS.

[Full marks may be obtained for about two-thirds of this paper. Great importance is attached to accuracy. N.B.— g may be taken = 32.]

1. Show how to find the resultant of a number of coplanar forces acting at a point.

Forces of magnitudes 3, 4, and 5, act at a point O in directions lying in one plane and making angles of 15° , 60° , and 135° respectively with a line OA in the same plane. Find to two places of decimals the magnitude of the resultant.

2. A small ring of weight W , which can move without friction on a circular wire fixed in a vertical plane, is in equilibrium at a point P on the lower half of the wire under the action of a force R in the direction of the tangent at P to the wire. If the pressure of the ring on the wire is equal to $\frac{1}{2}W$, find the magnitude and direction of the force R .

3. Define the centre of gravity of a rigid body. What assumptions with regard to the action of gravity are made for the purpose of the definition?

$ABCDE$ is a lamina of uniform thickness and density, and of such a shape that $BCDE$ is a square, and $AB = AE$. If the centre of gravity of the lamina is in BE , find the ratio of AB to BC .

4. Find the relation between the power and weight in the wheel and axle.

Show how to arrange three wheels and axles, having radii R and r respectively, so that $P/W = r^3/R^3$.

5. A body of weight 16 lbs. rests on a rough inclined plane inclined at an angle of 30° to the horizon. If a force of 2 lbs. acting up and parallel to the plane is just sufficient to prevent the body from slipping down, find the least force in the same direction which will balance the maximum resistance of the body to motion up the plane.

6. State the proposition known as the "parallelogram of velocities."

Prove that if a point possesses two independent velocities represented by $\lambda \cdot OA$ and $\mu \cdot OB$, where OA and OB are two straight lines meeting at O , the resultant velocity will be represented by $(\lambda + \mu)OG$, where G is a point on AB such that $\lambda \cdot AG = \mu \cdot GB$.

7. A person rows with a velocity of 6 miles an hour across a river a quarter of a mile wide, which runs with a velocity of 4 miles an hour. The head of the boat makes a constant angle θ with the bank while he rows across, and he arrives at a point 36 yds. 2 ft. lower down the bank than the point opposite his starting point. Prove that $\tan \theta = \frac{4}{3}$.

8. Prove that if a point moves from rest with a constant acceleration f , the distance s passed over in a time t is given by $s = \frac{1}{2}ft^2$.

A body falls from the top of a tower, and after 2 seconds another body is projected downwards with a velocity of 192 feet per second. The two bodies reach the ground at the same time. Find the height of the tower.

9. A balloon when at a height of $2021\frac{1}{2}$ feet from the ground begins to fall with a uniform acceleration of $\frac{1}{16}g$. When the balloon is at a height of $500\frac{1}{2}$ feet from the ground, ballast to the amount of one-tenth the whole mass of the balloon is thrown downwards with a velocity, relative to the balloon, of 10 feet per second. Find the time the ballast will take to reach the ground.

10. A particle is projected from a point P with velocity v in a direction making an angle α with the horizon. Prove that the greatest height above P , to which the particle rises, is $\frac{v^2 \sin^2 \alpha}{2g}$.

A stone is thrown from a height of 4 feet so as just to pass horizontally over a wall which is 25 yards distant and 54 feet high. Find the velocity and direction of projection.

ANSWERS.

JULY, 1885.

II. ARITHMETIC.

- | | | |
|---------------------------------------|--|---------------------------|
| 1. $18\frac{9}{10}$. | 2. .8. | 3. 3927 : 5000. |
| 4. '1418, 2'65. | 5. 19s. $1\frac{1}{2}d$. | 6. $13\frac{1}{2}$ acres. |
| 7. $5\frac{1}{4}$ lbs. | 8. '790625, 107'5625. | 9. 840. |
| 10. £4. 3s. 5d. | 11. $754\frac{1}{3}$ sq. ft. | 12. £1044. 12s. 1d. |
| 13. 8s. 5d. | 14. 4%. | 15. 10%. |
| 16. 50 tons 17 cwt. 3 qrs. 12 lbs. | 17. $15\frac{1}{2}$ past 24, 1 hr. $2\frac{1}{2}$ min. | |
| 18. £1234. 4s., £617. 2s., £205. 14s. | 19. $24\frac{4}{5}$ loss. | 20. £350. |
| 21. 2, 3, '1902751, 3'04077. | 22. $1\frac{1}{4}$ years. | |

III. ALGEBRA.

1. $\frac{x^5}{4} - \frac{11x^3}{24} + \frac{77x}{24} - \frac{23}{4}$; $16x^{\frac{4}{5}} - 8x^{\frac{3}{5}}y^{\frac{1}{5}} + 4x^{\frac{2}{5}}y - 2x^{\frac{1}{5}}y^{\frac{3}{5}} + y^2$.
2. $x^2 + x + 1$; $4(9x^2 - 4)(x^2 - 9)$.
3. 12.
4. (1) $\frac{3}{x(x^3 - 1)}$; (2) $\frac{(x-5)^2}{3x^2 - 32x + 64}$; (3) $\frac{x^2 - 4xy + y^2}{x^2 + y^2}$.
5. $3\sqrt{3} - 2\sqrt{5}$, 4.
6. (1) 3; (2) $\frac{3b}{2}$, $-\frac{a}{2}$; (3) $x = 0, \pm 3$; $y = \pm 2, \pm 1$.
7. 5 : 12.
8. $3x^2 - 2a^2x - a^4 = 0$.
9. $2\frac{1}{4}$ ft.
10. $2\frac{1}{7}$, $1\frac{1}{3}$, $1\frac{4}{11}$, $1\frac{2}{3}$, 1.
11. 455, $2(\sqrt{12})^2$.
12. Scale of 5.
13. $\frac{2n(2n-1)\dots(2n-r+2)}{r-1} a^{2n-r+1} x^{r-1}$, 3003.

IV. TRIGONOMETRY.

- | | |
|--|---|
| 1. $135^\circ = \frac{3\pi}{4}$. | 3. $n\pi + \frac{\pi}{4}$ and $n\pi + \frac{\pi}{3}$. |
| 8. $-\sqrt{1+\sin A}$, $+\sqrt{1-\sin A}$. | 8. 15° , 135° , or 45° , 105° . |
| 9. $132^\circ 34' 32''$. | 10. $11^\circ 157'$. |

WOOLWICH ENTRANCE EXAMINATION.

V. PURE MATHEMATICS (1).

1. Plane through line of intersection.
8. $-13\frac{1}{2}, 19\frac{1}{2}$.
9. Circle through point of intersection.
11. $\frac{x}{a}(\cos \phi - \sin \phi) + \frac{y}{b}(\cos \phi + \sin \phi) = 1$.
12. $\pm a\sqrt{m^2 - 1}$.
13. $(x - y)(\cos \theta - \sin \theta) = a(\cos \theta + \sin \theta)$.

VI. PURE MATHEMATICS (2).

1. $1 + \frac{n(n-1)}{1^2} + \frac{n(n-1)(n-2)(n-3)}{1^2 \cdot 2^2} + \frac{n(n-1)\dots(n-5)}{1^2 \cdot 2^2 \cdot 3^2} + \dots$
5. $\{x + x^2 - (2n+1)x^{n+1} + (2n-1)x^{n+2}\} \div (1-x)^2$.
6. $x^2 + y^2 - b^2 = a\sqrt{y^2 + (b+x)^2}$.
10. $(pq-r)x^3 - (p^3 - 2pq + 3r)x^2 + (pq - 3r)x + r = 0$.
11. 2, 2, 3.

VII. PURE MATHEMATICS (3).

1. $\frac{-2n(1-x)^{n-1}}{(1+x)^{n+1}}$.
3. $\frac{(-1)^{n-1} [n-1]}{x^n}$.
4. $\log 2 + \frac{1}{2}x + \frac{1}{8}x^2 + \dots$
6. $\frac{ab}{\sqrt{a^2 + b^2}}$.
7. Perpendicular to axis of x .
8. $\infty, \sqrt{8}$.
9. $xx'^3 + yy'^3 = a^4, \left(\frac{a^4}{a}\right)^{\frac{4}{3}} + \left(\frac{a^4}{\beta}\right)^{\frac{4}{3}} = 1$.
10. $\frac{2}{\sqrt{3}} \tan^{-1} \frac{2x+1}{\sqrt{3}}, \frac{1}{2}x + \frac{1}{4} \sin 2x, \frac{e^{ax}}{a}$.
11. $x \sin^{-1} x + \sqrt{1-x^2}$.
13. $\log 2$.

VIII. STATICS.

3. Force $4\sqrt{2}$, couple of moment $10 \times$ side.
5. $\frac{34}{15}$ of distance between the strings.
6. $r\sqrt{3}$.

IX. DYNAMICS.

1. $\frac{15v}{22}$.
2. $5\frac{5}{11}$ secs.
4. 6 secs.
5. $16\sqrt{5}$ f.s., 100 ft.
7. Very nearly $12\cdot3$.
10. $\frac{3}{2}P$.

NOVEMBER, 1885.

II. ARITHMETIC.

1. 20224372008, '0123402.
2. '0000790123456.
3. 294, 5'04.
4. £8. 3s. 6½d.
5. 7s. 0½d.
6. 300.
7. 5 cwt. 1 qr.
8. '3375, 1s. 2'75208d.
9. £10. 16s. 3'15d.
10. 7.
11. £8. 10s. 2'88d.
12. 3s. 4d.
13. £1. 1s.
14. 3 per cents., £3. 12s. 0½d.; 3½ per cents., £3. 10s. 9⅙d.;
4 per cents., £3. 14s. 5⅙d.
15. £6. 14s.
16. £14. 1s. 8'8125d.
17. Course 1800 yards, speed 9⅔ and 9⅓ miles per hour.
18. 420.
19. 270.
20. 2'947519, '947519, 2'947519.
21. '9542426, 2'1072100, '0030103, '3802113.
22. '23456.

III. ALGEBRA.

1. $3a - \frac{7}{2}b + \frac{1}{3}c$.
2. $2x^2 - 3x + 1$.
3. (i.) $\left(\frac{b}{a+b}\right)^3$; (ii.) $x^2 + y^2$; (iii.) 1.
4. $\frac{3a^m c}{2b^6} - a^n b^3 - \frac{2^8}{3}$, $3\sqrt{7} - 2\sqrt{6}$.
5. (i.) $\sqrt{3} \cdot ab$; (ii.) 7.
6. (i.) $\frac{ab}{a+b}$; (ii.) 1, $-3\frac{1}{2}$; 1, $-\frac{2}{7}$; (iii.) 0, 0, 0, and $\pm \frac{2}{\sqrt{35}}, \pm \frac{2\sqrt{5}}{\sqrt{7}}, \pm \frac{2\sqrt{7}}{\sqrt{5}}$.
7. 5, $3\frac{1}{2}$, $2\frac{3}{8}$.
8. $20\frac{1}{2}$, 16 feet.
10. 10 ft. 3 in.
11. $\frac{\beta^8 - a^8}{\beta - a}$.
13. $\frac{231}{4}x^{12}$.

IV. TRIGONOMETRY.

3. $\frac{1}{6}\left(n\pi \pm \frac{\pi}{6}\right)$, $\frac{1}{6}\left(2n+1\pi \pm \frac{\pi}{3}\right)$.
10. 1 mile, 1'219714 miles.
11. 25'7834.

V. PURE MATHEMATICS (I).

1. A straight line.
7. $a \cos \frac{\alpha - \beta}{2}$.
8. Straight line through the origin.

VI. PURE MATHEMATICS (2).

1. $1 + x + \frac{1}{2}(x^2 + x^3) + \frac{1 \cdot 3}{2^2}(x^4 + x^5) + \dots + \frac{1 \cdot 3 \dots (2n-3)}{2^{n-1}}(x^{2n-2} + x^{2n-1}) + \dots$
2. $\frac{n(n+1)(3n^2 - n - 2)}{24}$.
4. $0, \frac{3-\sqrt{5}}{2\sqrt{5}}, \frac{-3+\sqrt{5}}{2\sqrt{5}}$.
6. $-a, \pm\sqrt{-a^2}$.
7. $(a^2 - 2b)(b^2 - 2ac) - c^2$.
11. 2.
12. $\cos^2 A + \cos^2 B + \cos^2 C + 2 \cos A \cos B \cos C = 1$.

VII. PURE MATHEMATICS (3).

1. $\frac{2(1+x^2)}{(1-x^2)^2}, \frac{2}{1+x^2}, \frac{2(1+x^2)}{x(1-x^2)}$.
 2. $e^{3x}(3 \cos 4x - 4 \sin 4x), \frac{5^n x^n}{n} \cos(n\phi)$.
 3. $40, 8; n\pi + a, (2n+1)\frac{\pi}{2}$.
 4. $yy_1^2 + xx_1^2 = 2c^2$.
- The point $2, \frac{1}{10}$.
8. $0, \frac{13\sqrt{13} \cdot a}{6}$.

$$\log(x+1), \log \frac{x+1}{x+2}, \log(x+\sqrt{x^2+1}).$$

10. $\frac{e^x}{2}(\sin x + \cos x), \frac{x^2-1}{2} \log(x+1) - \frac{1}{4}(x-1)^2$.
11. $\frac{2}{\pi}$.
12. (1) $\frac{4a^2}{3}$; (2) $\frac{\pi a^2}{4}$.

VIII. STATICS.

1. $\sqrt{3} : 1 : 1$.
6. 55 lbs.
8. $\frac{W \sin 2A}{4 \sin A \sin B \sin C}, \frac{W \sin 2B}{4 \sin A \sin B \sin C}, \frac{W \sin 2C}{4 \sin A \sin B \sin C}$.
10. $2w$.
12. $\frac{\pi r}{4}$ from centre.

IX. DYNAMICS.

3. $15\frac{2}{5}$ lbs. per ton, 2 m. 572 yds.
4. 80 ft.
6. 9 lbs.-weight.
7. $4\cdot52, 3\cdot72$ secs.
8. $38\cdot90, 55$.
9. 28.
11. $25 : 4$.

ANSWERS.

JUNE, 1886.

II. ARITHMETIC.

1. One hundred thousand and two hundred millions, two hundred thousand, one hundred.
2. 90900, 1.
3. £730. 16s.
4. £5. 12s.
5. 19s. $1\frac{1}{2}d.$
6. 25.
7. 1418, 289.
8. £37. 7s. $7\frac{3}{8}\frac{1}{3}d.$
9. £13. 0s. 4d.
10. £6735.
11. £289.
12. 36 sq. ft. $74\frac{5}{8}$ in.
13. 1234 $\frac{2}{7}$.
14. At 5 hrs. 12 min.
15. £1. 19s. 4d. or $21\frac{8}{7}\frac{3}{7}$ p.c.
16. $176\frac{2}{3}\frac{7}{12}$.
17. 2'5984375, 6296'4...
18. 714285, £108.
19. 52 $\frac{1}{4}$.
20. 2'0897995.
21. 855677.

III. ALGEBRA.

1. (i.) $\frac{x+y}{x^2-xy+y^2}$; (ii.) 0. (iii.) $\frac{x^2-3}{(x-1)^2}$.
2. $x^2 + \sqrt{2} \cdot xy + y^2$, $x^2 + y^2 + x + y - xy + 1$.
3. $2xy$.
4. (i.) $\frac{a+b+c}{2} - \frac{b^2c^2+a^2c^2+a^2b^2}{4abc}$; (ii.) $\frac{1}{2}(-5 \pm \sqrt{19})$.
5. $acx^2 - (b^2 - 2ac)x + ac = 0$.
6. 5, 7.
7. 60 $\frac{1}{2}$ and 40 yds.
8. 1, 4, 7, etc.
9. $\frac{4}{13}$.
10. 0, $-\frac{1}{2}$.
11. 40320.

IV. TRIGONOMETRY.

3. $180^\circ + A$.
4. $\sqrt{2}$.
7. $65^\circ 59'$, $41^\circ 56' 12''$.

VI. PURE MATHEMATICS (2).

1. 4. 3. $1 + 5x + 19x^2 + 65x^3 + \dots + (3^{n+1} - 2^{n+1})x^n + \dots$
4. (1) $\frac{n(n+1)(n+2)}{3}$; (2) $\frac{n}{n+1}$, 1.
5. $x = 42, 31, 20, 9$; $y = 7, 16, 25, 34$. 6. 24.
8. (1) $2I$; (2) $\frac{6 \cdot 7 \cdot 8 \dots (5+n)}{n}$. 10. ∞ .
11. $2 \pm \sqrt{-3}, -6, 2$. 12. .88

VII. PURE MATHEMATICS (3).

1. $\frac{1}{1+x^2}$, cosec x , $x^x(\log x + 1)$.
4. $-10, \pm \frac{1}{16\sqrt{2} \cdot b} \{3a \pm \sqrt{a^2 + 32b^2}\} \sqrt{16b^2 - a^2 \pm a\sqrt{a^2 + 32b^2}}$.
5. $xx_1^{-\frac{1}{3}} + yy_1^{-\frac{1}{3}} = a^{\frac{2}{3}}$. 6. $\sqrt{\frac{2y^3}{a}}$. 7. $(a^2 + b^2) \frac{\pi}{4}$.
8. $y + x + a = 0, 2y + x = 2a, x + a = 0$.
9. $\frac{1}{8} \cos^8 x - \frac{1}{8} \cos^6 x, \frac{x^3(3 \log x - 1)}{9}, \frac{1}{8} \log \frac{x^3 - 1}{x^3 + 1}$.
10. $\frac{\alpha}{\sin \alpha}, \frac{4}{3}(\sqrt{2} - 1), \frac{\pi}{2} - 1$.
12. (1) $\frac{8a}{27} \left[\left(1 + \frac{9x}{4a} \right)^{\frac{3}{2}} \right]_{x_1}^{x_2}$; (2) $\left[\frac{3}{2} \sqrt[3]{ax^2} \right]_{x_1}^{x_2}$.

VIII. STATICS.

5. 1 lb. at distance 5 ft. 8. .2678... in. 9. $\frac{1}{4}$ altitude of triangle.

IX. DYNAMICS.

1. 15, 9 miles an hour. 2. 5 miles an hour.
4. Acc. 8, Tension $3\frac{3}{4}$ oz., Pressures 3, $2\frac{1}{2}$ oz. 7. 12 f.s.

NOVEMBER, 1886.

II. ARITHMETIC.

1. 7256.
2. £962.
4. 7 tons 6 cwt. 1 qr.
5. 1'34455, '01.
6. £962. 9s. $3\frac{3}{4}d.$
7. 5 days.
8. $113\frac{1}{2}$, 483.
9. £194. 12s. $7\frac{1}{2}d.$
10. £171. 1s. $10\frac{1}{2}d.$
11. £1333. 6s. 8d., $3\frac{7}{8}$ p.c.
12. 5 cub. yds. 25 ft. 1567 in.
13. 25 p.c.
14. 5 days.
15. 152'475 lbs.
16. 31 yds. 2 ft. 4 in., 3s. 9d.
17. 2825761, 2560000.
18. £3550, £3124, £2982; £3536, £3128, £2992.
19. 1s. $3\frac{1}{4}d.$ nearly.
20. '076923, £7. 9s. 4d.
21. (1) '8450980. (2) '3010299, '4771213.
22. 39'405.

III. ALGEBRA.

1. $2a^2b^2 + 2ab + 1, x^2 + 2 + x^{-2}.$
2. (i.) $\frac{1}{x^2 + y^2};$ (ii.) 0. (iii.) $\frac{x^2 + 2ax + 3a^2}{x^2 - 3ax + a^2}.$
4. (i.) $\frac{24a^2}{25b};$ (ii.) $x, \pm 3, \pm 1; y, \pm 2.$
5. 32 ft., 24 ft., 10 ft.
7. $5\frac{1}{2}, 6\frac{3}{8}$ hrs.
10. 8, and 2.
11. 68, 888.
12. 60.

IV. TRIGONOMETRY.

1. $65^\circ 27' 16\frac{4}{11}''.$
3. $\frac{\sqrt{3}-1}{2\sqrt{2}}, \frac{\sqrt{3}+1}{2\sqrt{2}}, -\frac{\sqrt{3}+1}{2\sqrt{2}}, \frac{\sqrt{3}-1}{\sqrt{3}+1}.$
5. $\frac{318}{888}.$
7. '22221.
9. $93^\circ 22' 21'', 47^\circ 41' 7'', 38^\circ 56' 32''.$
10. 2'39, 1'366 miles.

V. PURE MATHEMATICS (I)

3. The line of centres.
8. $\frac{a-A}{A\sqrt{a^2+b^2}}.$
11. $\frac{b^2\cos^2\theta - a^2\sin^2\theta}{ab\sin\theta\cos\theta}.$
12. (i.) Hyperbola, centre $-a, -b;$ (ii.) Ellipse, centre $\frac{1}{2}, 2.$
(iii.) A parabola and straight line.

VI. PURE MATHEMATICS (2).

4. $\lfloor 5, \lfloor n, \lfloor n-1.$ 6. $x^2 - 2x \cos 2\theta + 1 = 0.$ 7. $3, 3, \frac{3}{4}.$
 8. $1 - x + x^4 - x^5 + \dots + x^{4r} - x^{4r+1} + \dots$ 11. $\frac{a}{\sqrt{2}}(\cot \theta + 1).$

VII. PURE MATHEMATICS (3).

2. 71'4 yards. 3. 3'1390625.
 4. Triangle is a minimum when M bisects $PQ.$
 5. $\pm \frac{a^2}{\sqrt{a^2+b^2}}, \pm \frac{b^2}{\sqrt{a^2+b^2}}.$ 6. $\frac{y^2}{c}, c.$
 7. $p^2 = mr.$ 8. $y \pm \left(x + \frac{a}{2}\right) = 0, x - 2a = 0.$
 9. (1) $\log \frac{(x-4)^4}{(x-3)^3};$ (2) $\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}.$ 11. $\frac{a^2}{2}.$

VIII. STATICS.

2. $W.$ 3. 3 oz. 5. $\frac{45}{88}$ height.
 6. 8 oz., 1 lb. 10. 1000 ft. 11. 10'7.

IX. DYNAMICS.

1. At angle $\cos^{-1}\frac{3}{5}$ to the bank ; perpendicularly. 6. $7\frac{31}{100}$ miles.

ANSWERS.

JUNE, 1887.

II. ARITHMETIC.

1. 18.
2. £46. 5s.
3. $1\frac{3}{15}$.
4. £11. 11s. 3d.
5. £100.
6. £56. 15s. $4\frac{7}{8}$ d.
7. '000001645, '51.
8. 1s. $2\frac{3}{4}$ '00832d., '90044890873015.
9. 7963, 40'5.
10. $4\frac{1}{2}$.
11. 1s. 7'97184d.
12. Int.: Disc. = 5 : 4.
13. £19049. 3s. Diff. of income £253. 8s. 9d.
14. 12 sq. ft. 81 in., £12. 8s. $6\frac{1}{2}\frac{5}{12}$ d.
15. 2s. 11d.
16. 1007'6 lbs. (if packed in line 1018'3 lbs.).
17. $\frac{3}{65}$, '0000790123456.
18. 60 days, 12 hrs. 16 min.
19. 15'44.
20. $\bar{1}^{\circ}9488474$, '3344539, $\bar{3}^{\circ}7781513$; value = 6'240325; 19 and 20.
21. 10'08.

III. ALGEBRA.

1. $x^4 + x^3 - a^2x^3 + \frac{x^3}{a^2} + a^2x - x + 1, x^2 + (3+n)ax + 3a^2$.
2. $x^2 - x + 1$.
3. (1) $\frac{1}{x(x-a)(x-y)}$; (2) 2.
4. $\sqrt{\frac{3}{2}} - \sqrt{\frac{7}{2}}, a$.
6. (1) 3, 6. (2) 13 and $2\frac{1}{2}$. (3) $\frac{2a^3 + b^3 \pm b\sqrt{b^4 + 8a^3b - 4ab^3 + 4a^4}}{2(a^2 - b^2)}$.
8. $\frac{nb + (n+1)a}{2n+1}, \frac{na + (n+1)b}{2n+1}$.
9. Sum $\frac{1-3^{2n}}{4}$, last term -3^{2n-1} ; sum $-2n$, last term $1-4n$.
10. 1221759, 720.
11. 28, 3.
12. 3s. 4d. in the £. Land : personal property = 2 : 3.

IV. TRIGONOMETRY.

1. $\sin A = \frac{3}{5}, \cos A = \frac{4}{5}, \cot A = \frac{4}{3}, \sec A = \frac{5}{4}, \operatorname{cosec} A = \frac{5}{3}$.
7. $49^{\circ} 28' 32''$.
8. $\bar{1}^{\circ}93753, \bar{1}^{\circ}76143$.
9. 120° .
10. $\frac{2}{3}$.
11. Height = $\frac{25}{2}\sqrt{7} = 33^{\circ}07'189, CD = 17\frac{1}{2}$ ft.

WOOLWICH ENTRANCE EXAMINATION.

V. PURE MATHEMATICS (1).

2. A circle on line, drawn from pt. perpendicular to given line, as diameter.
 4. -14. 5. St. line. 7. $y = a \pm ex$.

VI. PURE MATHEMATICS (2).

1. $y^6 - 6y^4 + 9y^2 - 2$. 2. $A = C = 1, B = -D = \frac{1}{2\sqrt{2}}$.
 3. (1) $\frac{n}{3}(4n^2 + 6n - 1)$; (2) $\frac{1+x-(2n+1)x^n+(2n-1)x^{n+1}}{(1-x)^2}$.
 4. $\frac{1}{2}(a - a')$. 5. A little less than 36 years.
 7. $\frac{7}{120}$. 8. $\frac{\sin 2na}{2 \sin a}$.
 10. $\frac{e^\beta + e^{-\beta}}{2} \sin a, \frac{e^\beta - e^{-\beta}}{2} \cos a$; $\frac{2(e^\beta + e^{-\beta}) \cos a}{e^{2\beta} + e^{-2\beta} + 2 \cos 2\beta}, \frac{2(e^\beta - e^{-\beta}) \sin a}{e^{2\beta} + e^{-2\beta} + 2 \cos 2\beta}$.
 11. 4, $\pm \sqrt{3}$. 12. 2, 2, $\frac{1}{2}(-1 \pm \sqrt{-3})$.
 13. $rx^3 - (qr - 2q)x^2 + (pr^2 - 4pr + 4p)x - r^3 + 6r^2 - 12r + 8 = 0$.

VII. PURE MATHEMATICS (3).

1. $\frac{x}{x^4 - 1}, \frac{a^2}{(a^2 - x^2)^{\frac{3}{2}}}, (\sin x)^x \{ \log \sin x + x \cot x \}$.
 2. $2y \frac{d^3 y}{dx^3} + 6 \frac{d^2 y}{dx^2} \cdot \frac{dy}{dx}$. 4. $\frac{1}{2}x^2 + \frac{1}{3}x^3$. 5. $a^a(1 - \log a)$.
 6. $\frac{a\sqrt{m+n}}{\sqrt{mn}}$. 7. $\frac{ny}{y'} + \frac{mx}{x'} = m + n$.
 8. $x - 4 \log(1+x) - \frac{4}{1+x}, \log(2x+1+2\sqrt{1+x+x^2})$ or $\sin^{-1} \frac{2x-1}{\sqrt{5}}$,
 $\frac{x^r}{r^2}(r \log x - 1)$. 11. $\frac{n}{m-n}(m > n)$.

VIII. STATICS.

1. $6\frac{1}{2}$ lbs. 2. $\frac{1}{2}(W_2 + W_1), \frac{1}{2}(W_2 - W_1)$.
 3. $2\frac{4}{5}, 9\frac{2}{5}$ lbs. 4. $7P$.
 5. $3\frac{1}{2}$ lbs., $\frac{1}{2}$ in. from centre. 7. $2\sqrt{5}, 3$ inches.
 8. 3808 ft.-lbs. 10. $1\frac{1}{3}$ lbs.

IX. DYNAMICS.

1. 2 f.s.s.
2. $v \cot \theta$.
3. Accel. = 4*f*. $L = 32T^2$ (L ft. = unit of length, T secs. = unit of time).
5. A circle.
8. A parabola.

NOVEMBER, 1887.

II. ARITHMETIC.

1. 899910.
2. £19. 15*s*. 6*d*.
3. $4\frac{6}{81}$.
5. '00212, 21200.
6. 2'7765625, 2 qrs. 7 lbs.
7. $\frac{288}{37000}$, 8'54854470131697.
8. £1. 16*s*.
9. £150. 10*s*. $5\frac{1}{2}$ *d*.
10. 14'0700007.
11. 8 cub. ft. $656\frac{3}{8}$ in.
12. 12 years.
13. £3.
14. 120 days.
15. $16\frac{6}{199}$.
16. $34\frac{4}{9}$.
17. 4, $\frac{1}{2}$: 5'3692159, 1'3692159, 2'3692159.
18. '0189694.
19. 367182.
20. (1) £6000; (2) £131501. 5*s*. 5*d*.
21. 1 min. $52\frac{1}{2}$ secs. after the dog started.
22. £819. 6*s*. 2'67*d*.

III. ALGEBRA.

1. $x(a+1)-y(b-1)$.
3. $x^2+7xy-y^2$.
4. (1) 2; (2) $-\frac{34xy}{18x^2-y^2}$.
6. $2\sqrt[3]{\frac{a}{c}}+2\sqrt[3]{\frac{c}{a}}-1$.
8. (1) $6a+b$, $2a-b$; (2) $3a$ and $\frac{3a}{2}$; (3) $-\frac{5}{4}$.
9. 1020 yards.
10. 180.
11. 210.
12. $(-1)^n \frac{2n}{(n^2)^3} \cdot \frac{1}{6}(r+1)(r+2)(r+3)$.

IV. TRIGONOMETRY.

4. $\frac{\pi}{3}$, $\frac{1-\sqrt{3}}{2\sqrt{2}}$.
6. $\frac{4 \tan A(1-\tan^2 A)}{(1+\tan^2 A)^2}$, $n\pi + \frac{\pi}{4}$, $\frac{n\pi}{2} + \frac{3\pi}{8}$.
8. 20'97615.

WOOLWICH ENTRANCE EXAMINATION.

V. PURE MATHEMATICS. (1)

4. $\frac{1}{2}\sqrt{\{b^2c^2 + a^2c^2 + a^2b^2\}}$. 5. A parabola. 6. $\frac{a^2}{\sqrt{a^2+b^2}}, \frac{b^2}{\sqrt{a^2+b^2}}$.
 7. Rect. hyperbola, $x^2 - y^2 = c^2$, when origin is mid-pt. of AB , and $AB = 2c$.
 10. $xx' + yy' = 0$. (1) A pt. at the origin. (2) 3 st. lines—axis of y , $x - y = 0$ and $x + y = 0$. (3) Four points $x = \pm a$, $y = \pm b$.

VI. PURE MATHEMATICS (2).

3. $y = 1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{1}{16}x^6 - \dots = x + \frac{1}{2x} - \frac{1}{8x^3} + \frac{1}{16x^5} - \dots$
 5. $\frac{1}{2}n(n-1)$, $2n(n-1)$. 6. $\frac{109}{25}$. 8. 2, 2, $-\frac{1}{2}$.
 9. $(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{3}{2}}$. 10. $\frac{\sin \theta}{1 - 2x \cos \theta + x^2}$.
 11. 0, $\tan \frac{\pi}{n}$, $\tan \frac{2\pi}{n}$, $\tan \frac{3\pi}{n}$... $\tan \frac{(n-1)\pi}{n}$. 12. $\frac{d^2 + 2ed}{(e+d) \cot \theta - e \cot \phi}$.

VII. PURE MATHEMATICS (3).

3. $\frac{ab(1 + \tan^2 y)}{a^2 + b^2 \tan^2 y}$. 6. $\frac{2x(x-a)}{a}$; $\frac{a}{2}$, $\pm a$.
 7. $r^n = a^n \cos n\theta$ where $n = \frac{m}{m+1}$. 8. $y = \pm x + \frac{a}{2}$, $\pm \frac{\pi}{6}$ with axis of x .
 9. $\frac{1}{2}x - \frac{1}{4} \sin 2x$, $x \log x - x$, $\frac{1}{6} \log \frac{x-2}{x+3}$.
 10. 1, 1, $\frac{\pi}{3\sqrt{3}}$. 11. $\frac{a^2\pi}{12}$. 12. $\frac{8a}{27} \left(1 + \frac{9x}{4a}\right)^{\frac{3}{2}} - \frac{8a}{27}$.

VIII. STATICS.

2. 14'242. 3. $6\frac{1}{4}$, $11\frac{3}{8}$ lbs. 4. 14'56 lbs.
 7. $\frac{arW}{(a+b)\sqrt{r^2-ab}}$, $\frac{brW}{(a+b)\sqrt{r^2-ab}}$. 8. $5\frac{8}{11}$ in.

IX. DYNAMICS.

1. 1 : 20 : 360. 4. 115200 feet. 5. $2\frac{1}{2}$, $3\frac{1}{8}$ lbs.
 9. 1'713 oz. 10. $5\frac{2}{7}$ tons.

ANSWERS.

JUNE, 1888.

II. ARITHMETIC.

- | | |
|---------------------------------|---|
| 1. 4623. | 2. $1\frac{7}{15}$. |
| 3. 149, 1317160. | 4. '01405609352, '90892. |
| 5. £42. 13s. 4d. | 6. $\frac{81}{195}$, $11\frac{611}{1950}$, $\frac{17287}{195000}$. |
| 7. $\frac{9}{100}$, '96510416. | 8. 14'13; 31. |
| 9. 8 days. | 10. £10. |
| 11. $4\frac{1}{2}$. | 12. £5. 19s. 2d. |
| 13. £1. 17s. 4d. | 14. £13. 18s. |
| 16. 6, -6, $\frac{5}{8}$. | 17. 4'5529844, 2'5529897. |
| 18. 83570'6. | 19. 198'5365. |
| 20. 38'7. | 21. 8'82888. |
| | 22. 2 miles or 24 min. after. |

III. ALGEBRA.

- | | |
|---|---|
| 1. 0. | 2. $2(a+b)(b+c)$, $5xy(x-y)(x^2-xy+y^2)$. |
| 3. $\frac{2x^2+3x-1}{x^2-3x-2}$. | 4. (i.) $\frac{34xy}{49x^2-y^2}$; (ii.) $-\frac{1}{abc}$. |
| 5. $\frac{a^2}{4} + \frac{2c}{a}$, $\frac{c^2}{a^2}$. | 7. (a) -20; (b) 5; (c) a and b. |
| 8. $\frac{7}{18}$. | 9. $r(\sqrt[3]{2}-1) = '259921r$. |
| 10. $2a$, $\frac{a}{2}$. | 12. For $n+r$ put $n+r-1$. |

IV. TRIGONOMETRY.

- | | |
|---|---|
| 1. $57^\circ 16' 21\frac{9}{11}''$. | 3. $\frac{1}{2}n\pi \pm \frac{\pi}{12}$. |
| 4. $\cos a + \cos 2a + \cos 3a + \cos 4a$. | 5. $\frac{5}{9}$. |
| | 8. 39 : 33 : 25 |
| 10. $A = 105^\circ 27' 35''$, $B = 15^\circ 5' 20''$, $c = 2'44844$. | 11. 45° . |

V. PURE MATHEMATICS (I).

9. $\tan^{-1}\frac{10}{11}$. 10. A parabola with same vertex and $\frac{1}{3}$ th the parameter.
 12. (i.) Axis of y and a hyperbola; (ii.) 2 st. lines; (iii.) A parabola.

VI. PURE MATHEMATICS (2).

1. $\frac{(a-b)(a-c)}{a^2+b^2+c^2-ab-ac-bc}$
 $\frac{(b-a)(b-c)}{a^2+b^2+c^2-ab-ac-bc}, \frac{(c-a)(c-b)}{a^2+b^2+c^2-ab-ac-bc}$
2. (i.) $\frac{2}{3}(4n^2+6n-1)$; (ii.) $\frac{1}{2}$. 3. $-2x^2+2x^3-4x^4+6x^5$.
4. $\frac{4}{\sqrt{3}}$. 6. 78.
7. $-2, \frac{1}{2}(1 \pm \sqrt{-11}); \pm\sqrt{2}, \frac{1}{2}(1 \pm \sqrt{-11})$.
8. (i.) $a, -\frac{a}{2} \pm \frac{\sqrt{16-3a^2}}{2}$; (ii.) $1, 1, \frac{1}{4}(-1 \pm \sqrt{-15})$.

VII. PURE MATHEMATICS (3).

1. $\frac{1}{\sqrt{1-x^2}}, \frac{\sqrt{x+a}}{x\sqrt{x-a}}$. 4. Max. 1 , min. $-\frac{1}{6}$; max. $\frac{1}{e}$.
6. $\frac{mx}{x'} + \frac{ny}{y'} = m+n$. 7. $\frac{(a^4y^2+b^4x^2)^{\frac{3}{2}}}{a^4b^4}$.
9. $\sin^{-1}\frac{2x-1}{\sqrt{5}}, \frac{1}{3}\cos^3x - \cos x, \frac{x^2}{2}\log x - \frac{1}{2}x^2$.
10. $\frac{\pi}{4}, n\pi, \frac{\pi}{3}$. 11. (1) πa^2 ; (2) $\frac{2}{3}\pi ab$.
12. $\sqrt{ax+x^2} + a \log \frac{\sqrt{x} + \sqrt{a+x}}{\sqrt{a}}$.

VIII. STATICS.

2. $\frac{1}{2}$ lb. 3. $2AD$. 6. $\frac{W_2}{W_1+2W_2} \cdot BC$ from the hinge.

IX. DYNAMICS.

2. $\frac{W}{32}$. 3. $\frac{D-d}{D} \cdot V$. 7. $\mu g \cos \alpha$.
8. 68.033 miles per hour. 9. 80000 feet.

NOVEMBER, 1888.

II. ALGEBRA.

1. 1.
2. $4x^2 - 6x + 9, (2x + 3)(16x^4 + 36x^2 + 81)(6x^2 - 5x - 6).$
3. $2\frac{x}{a} - 3 + 4\frac{a}{x}$
4. $b^2 > 4ac.$
5. (1) $1\frac{1}{2}$; (2) $a + b$, and $\frac{1}{2}(a + b)$; (3) 9, 6, 4 or 4, 6, 9.
6. 3, 8 gallons.
7. 15.
8. $\frac{2m+1}{m(m+1)}\left(x + \frac{1}{x}\right).$
9. Slightly over 125 years.

III. TRIGONOMETRY AND MENSURATION.

1. 21,600, 6875'49.
2. $2n\pi - \frac{\pi}{4}.$
3. $A = 22^\circ 37' 11\frac{1}{2}''$, $B = 67^\circ 22' 48\frac{1}{2}''$, $C = 90^\circ$. Area = 120 sq. ft.
4. 418'4, 430 ft.
5. 233'509 ft, 238'978 tons.
6. 11. 165'748.

IV. STATICS AND DYNAMICS.

1. Yes, arranged in same line.
2. $2P.$
3. 1 ft. from centre of lever.
4. $\frac{l \sin \alpha}{1 + \sin \alpha}$ from free end (α = inclination of plane to horizon).
5. 128.
6. $2\frac{1}{2}$ secs., $\frac{3}{4}$ weight.
7. 80 f.s., $80\sqrt{3}$ f.s.
8. $\frac{v\sqrt{h}}{4}.$

V. PURE MATHEMATICS (1).

1. $x - y = 0.$
2. $(\beta - ma - b)^2 = k^2(1 + m^2).$
3. $(c + ak^2)(c + bk^2) = h^2k^4.$
4. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$ Eccentric angle.
5. (1) Parabola; (2) Hyperbola; (3) Ellipse.

VI. PURE MATHEMATICS (2)

1. $\pm \frac{1 \pm \sqrt{5}}{2}$.
2. 2160 lbs. weight.
3. 12, -60, 60.
5. 19, 1, 80.
6. $-\frac{1}{8}x^4 - \frac{11}{8}x^5 - \frac{85}{216}x^6 - \dots - \frac{1}{2} \left(1 - \frac{1}{2^{n-4}} + \frac{1}{3^{n-3}} \right) x^n \dots$
8. $27^\circ 15'$.
9. 3.

VII. MECHANICS.

1. Straight line through point of intersection.
 2. Wt. 3.76 lbs. ; Tensions, 1.88, 1.28, 2.53 lbs.
 3. $54\frac{1}{2}^\circ, 47\frac{1}{2}^\circ, 78^\circ$.
 4. Force is $> W \tan \alpha$; $\frac{W \tan \alpha}{W + W'}$, (W' = weight of plane).
 6. μg . A st. line.
 7. $\frac{P-Q}{P+Q} \cdot h$.
 8. $\frac{\pi}{43200}$.
 10. $\frac{1}{2}v$, perpendicular to and $\frac{\sqrt{3}}{2}v$ along the line of centres ; st. line at angle of 30° to line of centres.
 11. $\frac{5}{8}$ ft.-lb.
-

ANSWERS.

JUNE, 1889.

II. ALGEBRA.

1. $4(a+c)(a-c)(x-z)(x+y+z)$. 2. $a+4$.
3. $\frac{1}{x^2-2xy-3y^2}$; $\frac{x+3y}{8xy(x+y)}$. 4. 30. 5. $\frac{a+b}{a-b}$; 36.
6. (i.) 6. (ii.) $x=-1$, $y=1$, $z=1$. (iii.) 0, -4.
7. (i.) p . (ii.) p^2-2q . (iii.) p^3-3pq .
8. The number of sovs., half-sovs., florins and half-crowns are proportional to 1, 3, 5, 6. 9. 0.
11. $1+2x+5x^2+\frac{40}{3}x^3+\frac{110}{3}x^4+\dots$
12. $\bar{3}^{\cdot}6020600$; $3^{\cdot}3219280$; 20.

III. TRIGONOMETRY.

1. $68^{\cdot}755$ in.; $17188^{\cdot}7$ ft. 2. $\alpha - \frac{\pi}{2}$; $\frac{1}{3}$, $\frac{5}{3}$.
3. $4 \operatorname{cosec} 2\theta$. 4. $\frac{1}{4}\{n\pi \pm \sqrt{16+n^2\pi^2}\}$.
5. $2 \sin A = -\sqrt{1+\sin 2A} + \sqrt{1-\sin 2A}$.
6. (i.) $m\pi$, and $\frac{m\pi}{n-1} - (-1)^m \frac{\pi}{6(n-1)}$. (ii.) $\sqrt{3}$.
8. $200^{\cdot}017$ ft. 9. $17^{\cdot}1064$ sq. in.
10. $960^{\cdot}08$ sq. ft. 11. 11 cwt. 1 lb. $1^{\cdot}32$ oz.

IV. STATICS AND DYNAMICS.

1. The angles between their directions are 150° , 120° , and 90° .
3. $\frac{5}{8}$, $\frac{7}{8}$ cwt. 4. $1\frac{1}{3}$ cwt.
5. $2:1:1$. 6. 12 lbs.; pressure 20 lbs.
7. Velocity 13, at an angle $\sin^{-1}\frac{11}{13}$ to the vertical.
9. 144 ft. 10. 60 yds.
12. At a point $2h$ from the foot of the tower, with velocity $2\sqrt{gh}$ f. s.

WOOLWICH ENTRANCE EXAMINATION.

V. PURE MATHEMATICS (1).

7. $\frac{a(b-b')}{b+b'}, \frac{2bb'}{b+b'}$. 8. $x^2+y^2=a^2$.
9. $y^2=4(a'-a)(x-a)$; if the given parabolas are $y^2=4ax$ and $y^2=-4b(x+c)$, the locus is $y^2=\frac{8ab}{b-a}\left(x+\frac{c}{2}\right)$.
11. (i.) A circle. (ii.) An ellipse, centre $a, -a$, semiaxes $\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{3}}$.
(iii.) A hyperbola, semiaxes $2a, 2b$.

VI. PURE MATHEMATICS (2).

1. y^3-3y, y^5-5y^3+5y . 2. $x=\pm 2, \pm \frac{1}{\sqrt{2}}; y=\pm 3, \pm \frac{5}{\sqrt{2}}$.
3. $-1 \pm 2\sqrt{-1}, 1 \pm \sqrt{2}$.
4. $y^3+(4bd-3c^2-ae)y-2b^2e-2c^3+2ace-2ad^2+4bcd=0$.
5. $\frac{2}{x-1}-\frac{3}{x+1}+\frac{x+3}{x^2-x+1}$. 6. $\frac{1}{2}(a-a')$.
7. (i.) $\frac{n\pi}{4}, \frac{2n\pi}{3} \pm \frac{\pi}{9}$. (ii.) $2n\pi \pm \cos^{-1} \frac{\pm \sqrt{17}-3}{4}$.
8. $\sin^2 \frac{n\theta}{2} \div \sin \frac{\theta}{2}$.
10. If $x=a \cos \alpha, y=a \sin \alpha, x'=b \cos \beta, y'=b \sin \beta$,

$$\tan \phi = \frac{a^m \sin ma + b^n \sin n\beta}{a^m \cos ma + b^n \cos n\beta},$$
 and $R = \{a^{2m} + b^{2n} + 2a^m b^n \cos(m\alpha - n\beta)\}^{\frac{1}{2}}$.
11. $B_1 = -\frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0, B_4 = -\frac{1}{36}$.

VII. MECHANICS.

2. 31.6 lbs.; $17^\circ 3'$. 3. 1. 5. $\frac{P-Q}{2}$.
6. 2658.3124, 658.3124 ft.-lbs. 7. 1 sec.; $1\frac{1}{2}$ secs. after first started.
8. The radius makes an angle $\cos^{-1} \frac{1}{4}$ with vertical.
10. $x^2 = \frac{V^2}{f} \cdot y$.

NOVEMBER, 1889.

II. ALGEBRA.

1. $x^2 + y^2 + z^2 - yz - xz - xy$; 0.
2. $3ax^2 - 2a^2x + a^3$.
4. $\{(b+c)x - ay\}\{ax + (b-c)y\}$.
5. $\sqrt{2x-3} + \sqrt{x+2}$.
6. $\frac{5}{24}$.
7. (i.) 4. (ii.) $\pm \frac{24a^2}{25b}$. (iii.) $x=0, \pm 3; y=\pm 2, \pm 1$.
8. 90, 110.
10. 105.
11. $(r+1)(2r+1); \frac{1}{2}(r+1)(r+2); 2$.
12. 2'5440680; '9116518.

III. TRIGONOMETRY.

1. $\frac{\sqrt{3}}{2}$.
2. $\sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5}, \text{versin } \theta = \frac{1}{5}$.
4. $n\pi, n\pi \pm \tan^{-1}(\sqrt{5})$.
5. 4.
6. 2.
7. $B = 110^\circ 48' 15'', C = 26^\circ 56' 15'', a = 93.5192$.
8. Height, 159.422 ft.; distance, 215.676 ft.
9. Area, 101543 $\frac{1}{2}$ sq. ft.; perimeter, 1129.614 ft.
10. $\frac{4}{3}$ diameter; at the top of the cylinder.

IV. STATICS AND DYNAMICS.

1. 60° .
3. A is 7 in. from nearest peg.
4. 10.14 ft.-lbs.
6. In the vertical part $2\frac{1}{4}$ in. from the junction.
7. 11 min.
8. (i.) $\frac{2}{3}\frac{5}{2}$ sec. (ii.) $\frac{5}{8}$ sec., and on return after $2\frac{1}{2}$ secs.
10. $53\frac{3}{4}\frac{5}{11}$ f. s.
11. $4\sqrt{3}$ f. s.

V. PURE MATHEMATICS (I).

7. $\frac{1}{2}p_1p_2\sin(a_1 - a_2)$.
9. $y = mx - 2am - am^3$.

VI. PURE MATHEMATICS (2).

1. $\frac{2a+2c-b}{3} \pm \frac{2}{3}\sqrt{a^2+b^2+c^2-bc-ac-ab}.$
2. 102.
3. 35; 24.
4. $\frac{16}{(\lfloor 4 \rfloor)^4} = 63063000.$
5. $A = \frac{(m-n)a+2nb}{m+n}, B = \frac{(n-m)b+2ma}{m+n}.$
6. $\sqrt{\frac{c(m+n)}{mn}}.$
7. $2n\pi + \alpha, 2n\pi + \frac{\pi}{2} - \alpha.$
8. $\left(x^2 + 2x \cos \frac{\theta}{2} + 1\right) \left(x^2 - 2x \cos \frac{\theta}{2} + 1\right).$
10. $\frac{\theta}{2}, \frac{\pi - \theta}{2}.$

VII. MECHANICS.

1. 3'4, 15'6 lbs.
 2. 2'15, 1'49 lbs.
 3. $2\frac{1}{3}\frac{3}{4}, 5'19$ oz.
 5. $\frac{1}{4}W.$
 6. $1400\sqrt{3}$ ft.-lbs.
 7. 4 secs. after the first started at distance 16 ft. from $O.$
 9. 31 : 33.
 10. $\frac{1}{8}$ of perimeter from point of projection.
 11. 32'249.
-

ANSWERS.

JUNE, 1890.

II. ALGEBRA.

1. $x^3 - 5x^2y + 6xy^2 - y^3$.
2. $x - 3a$.
3. 248.
4. (i.) 3; (ii.) $x = \frac{1}{2}(a+b)$, $y = \frac{1}{2}(a-b)$.
5. $9x^2 - 30x + 34 = 0$.
6. (i.) $2\frac{2}{3}$, $2\frac{2}{3}$; (ii.) $x = 2$, $r = \frac{2}{3}$; $y = 5$, 6.
7. $5(x^2 + y^2 + z^2 + yz + xz + xy)$.
8. $\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{7}$, $\frac{2}{9}$, $\frac{2}{11}$.
9. $2(n-1)(n-2)\dots(n-r+2)$.
10. $\frac{n(n+1)\dots(n+r-1)}{r!}$; $\frac{1 \cdot 3 \cdot 5 \dots (2r-1)}{2^r r!}$.
11. 2'1177312.
12. 15'468....

III. TRIGONOMETRY.

1. $57^\circ 17' 45''$.
2. $2 \cos A = -\sqrt{1 + \sin 2A} + \sqrt{1 - \sin 2A}$.
3. $ab \div (\sqrt{b^2 - 1} + \sqrt{a^2 - 1})$.
4. '1736 and '9848 of a mile.
5. $A = 83^\circ 7' 39''$, $B = 42^\circ 16' 21''$, $C = 199^\circ 099$.
6. $4\pi r^2$, $2\pi r h$, $\pi r l$ (l = slant side); 22'66 sq. yds.
7. 2'7373 in. Height 5'6696 in., base diameter 4'4097 in., and top diameter 3'4488 in.

IV. STATICS AND DYNAMICS.

1. $5\frac{1}{8}$, 2, 6, $1\frac{1}{8}$, $4\frac{1}{8}$ lbs. respectively.
2. Each = $\frac{W}{\sqrt{3}}$.
3. Taking 5 in. to 2 lbs., moment = $\frac{1}{8}$ ft.-lbs.
4. 1 lb., $4\frac{1}{2}$ in. from A.
5. 1 : $\sqrt{3}$.
6. $\frac{2}{3}\sqrt{3}$ miles an hour.
7. 25 ft.
8. $\frac{2}{3}\frac{1}{2}$ lbs.-wt.; $13\frac{1}{2}$.
9. 121 : 14400.
10. $240\sqrt{2}$ f. s.

V. PURE MATHEMATICS (1).

1. A circle.
2. A cylinder.
3. (i.) 4 points; (ii.) 2 pairs of lines parallel to the axes; (iii.) two parabolas, vertices at origin; (iv.) hyperbola.
4. A straight line.
5. $x^2 + y^2 - 2ax - 2by = 0$.
6. $y^2 = x^2 + 6ax + a^2$.

WOOLWICH ENTRANCE EXAMINATION.

VI. PURE MATHEMATICS (2).

2. (i.) 333667; (ii.) 200000. 3. 3125. 5. 56 and 44.
8. $(2n+1)\frac{\pi}{8}$, $\frac{2}{3}n\pi \pm \frac{\pi}{9}$. The angles are $202\frac{1}{2}^\circ$, 220° , $247\frac{1}{2}^\circ$, 260° , $292\frac{1}{2}^\circ$, $337\frac{1}{2}^\circ$, 340° .
9. $\left(x - \frac{1-\sqrt{-3}}{2} \cdot y\right) \left(x - \frac{1+\sqrt{-3}}{2} \cdot y\right)$. 10. $\frac{1}{3}a$, $\frac{2}{3}a$.
11. (ii.) $\frac{n}{2} - \frac{\cos(2\theta + n-1\alpha)\sin n\alpha}{2\sin \alpha}$.

VII. MECHANICS.

2. 3·9, 2·6 lbs. 3. 26·8 lbs.
4. $\sqrt{\{P^2 + Q^2 + R^2 + 2PR \cos \alpha + 2QR \cos \beta + 2PQ \cos(\alpha + \beta)\}}$;
 $\cot^{-1} \frac{R + P \cos \alpha + Q \cos \beta}{P \sin \alpha - Q \sin \beta}$ with direction of R .
5. $\frac{Wr}{2\sqrt{R^2 - r^2}}$. 7. 2 secs., and $5\frac{1}{2}$ secs.
8. $\frac{3WP}{W+4P}$. 9. $7x + 23y = 0$.
-

NOVEMBER, 1890.

II. ALGEBRA.

1. $x^6 - 4x^4 + 2x^3 - x^2 + 1$; $x - 2$.
2. $1 + 2abc - a^2 - b^2 - c^2$; $(x-y)^2 + (y-z)^2 + (z-x)^2$.
3. 1000, 1. 4. $x^2 - 1$. 5. (i.) 4; (ii.) $x = 1$, $y = -1$.
6. $q^2 > 4pr$; -2 and 8 . 7. (i.) a^2 , -1 ; (ii.) $x = 2$, 3 ; $y = 1$, 0 .
8. A 18, B 12 miles a day. 9. $\frac{26}{111}$. 10. 78.
11. $1 + 4x + 10x^2 + 20x^3 + 35x^4 + 56x^5 + \dots$; 2 .
12. (i.) 3'32'19278; (ii.) 18 years (17'67).

III. TRIGONOMETRY.

1. $\sin 210^\circ = -\frac{1}{2}$, $\sin 750^\circ = \frac{1}{2}$. 4. $\frac{12\pi}{8}$, $\frac{22\pi}{3} \pm \frac{\pi}{9}$.
7. -8 , $\frac{1}{4}$. 8. $40^\circ 56' 12''$.
9. Height 46'1402 ft., distance 99'92 ft.
10. 133'785 ft. 11. $\frac{1375}{4704}$ ton.

IV. STATICS AND DYNAMICS.

3. $\frac{1}{3}W$, $\frac{2}{3}W$. 4. $2\frac{5}{12}$ lbs., making an angle $\tan^{-1} \frac{2}{3}$ with BC .
5. For 6 lbs., 3 sheaves in each, string fixed to top block; for 7 lbs., 4 sheaves in top and 3 in bottom block, string fixed to lower block.
6. From any point distant $\frac{1}{3}$ side from an angle of the triangle.
7. 484 : 315. 9. $2\frac{2}{3}$ secs.; 64 f. s.
10. $42\frac{1}{4}$ poundals, or $1\frac{1}{128}$ lbs.-wt.

V. MATHEMATICS (I).

1. (i.) Circles; (ii.) Straight lines.
7. (i.) Two straight lines at right angles; (ii.) A circle, centre $\frac{3}{2}$, 2, and radius $2\frac{1}{2}$; (iii.) A parabola, vertex $-a$, $2a$; (iv.) An ellipse centre 6, 4, semiaxes 10, 5.
8. $y - y' = \tan a(x - x')$. Taking the fixed lines as axes, the hyperbola is $2xy = (x'y + xy')$, and the asymptotes are $2x - x' = 0$, $2y - y' = 0$.
10. $x^2 - 2ax^2 - ay^2 = 0$.

VI. MATHEMATICS (2).

2. $\frac{1}{a}, a^3, a^3.$
3. $1+x-x^3-x^4+x^6+x^7-\dots; 1+x+3x^2+5x^3+11x^4+21x^5+\dots$
5. $1-x-2x^2; 2^n+(-1)^n.$ 6. $385; \frac{1}{2}(9\sqrt{2}+2)$ diameters.
7. $\frac{n\pi}{2}, \frac{n\pi}{2} \pm \frac{\pi}{12}.$ 8. $d_1 = \frac{a \sin B \sin C}{\sin \frac{A}{2}(\sin B + \sin C)}.$ 10. $\sin^{-1} \frac{5}{13}.$
11. (i.) $\sin \frac{n-1}{n} \cdot \frac{\pi}{2} \div \sin \frac{\pi}{2n};$ (ii.) $\frac{x \sin \frac{\pi}{n} + x^{n+1} \sin \frac{n-1}{n} \pi}{1 - 2x \cos \frac{\pi}{n} + x^2}.$

VII. MECHANICS.

1. $4\frac{1}{12}$ lbs. acting in AC produced at distance 1'66 in. from C .
2. Tension, $47\frac{3}{11}$ lbs.; pressures $43\frac{7}{11}, 18\frac{9}{11}$ lbs.
3. $\frac{aW^2}{P^2}, \frac{2aW}{P}.$ 5. $\frac{4}{9}W, \frac{1}{6}W, \frac{7}{18}W.$
7. 4000 ft.-cwt. 8. $u + \frac{1}{2}f(2n-1)$ feet.
9. $\frac{Pg}{P+Q}$ f. s.; $\frac{PQ}{P+Q}$ lbs.-wt. 11. $50\frac{1}{11}.$

ANSWERS.

JUNE, 1891.

II. ALGEBRA.

1. $(x-1)^4 - (x-1)^2 + 1$.
2. $\frac{x^2+1}{x^2+2}$; $(x-14)(x-12)$, $(x^2+xy+y^2)(x^2-xy+y^2)$.
4. $2x^2+3x+4$. 5. (i.) $\frac{5}{7}$. (ii.) $1\frac{1}{2}$, $\frac{5}{8}$.
6. 654321, 1562202. 7. (i.) $\frac{b^2}{a}$, $-\frac{1}{a}$. (ii.) $x = \frac{3}{2}$, $-\frac{1}{2}$; $y = -\frac{1}{2}$, $\frac{3}{2}$.
8. $6\frac{1}{2}$ and 5 sq. ft. 10. $1 + \frac{1}{3} + \frac{1}{9} + \dots$; 10, 12, ... 60, infinity.
11. m , 5th and 6th terms = $\frac{4812560}{2560000}$. 12. 4'60898, £100,000.

III. TRIGONOMETRY.

1. $\frac{13}{12}$, $\pm \frac{\sqrt{13}}{3}$, $\frac{169}{120}$. 3. $\tan(n-1)A$.
8. $56^\circ 30' 56''$ and $123^\circ 29' 4''$. 9. $\sqrt{2(x+1)}$.
10. £19962. 13s. 6d. 11. 1221'88 cubic inches.

IV. STATICS AND DYNAMICS.

2. $\frac{1}{2}W$, $\frac{\sqrt{3}}{2}W$. 3. 7 oz., $9\frac{1}{2}$ inches from A .
4. $\frac{39+4\sqrt{6}}{75}a = .6506a$. 5. Round C in direction BOA .
7. $34\frac{2}{3}$. 8. $\frac{2}{3}$ f.s.s.
10. $\frac{1}{4}$ sec. 11. $91\frac{3}{8}$ f.s. 12. 350 ft.

V. PURE MATHEMATICS (1).

2. SP = abscissa, or latus rectum.
8. $x(a \sin \alpha + b \sin \beta) - y(a \cos \alpha + b \cos \beta) = 0$,
 $x(a \sin \alpha - b \sin \beta) - y(a \cos \alpha - b \cos \beta) = ab \sin(\alpha - \beta)$.
12. $\theta = \frac{\pi}{2}$, $r \cos \theta = \frac{2eL}{1-e^2}$.

NOVEMBER, 1891.

II. ALGEBRA.

1. 0.
2. $2x^7 - 7x^2 + 5, a^4 - 3a^3b + 6a^2b^2 - 3ab^3 + b^4.$
3. $x - 1, x^{15} + 2x^{14} + 3x^{13} + 4x^{12} + \dots$
4. $x^2 + 2x + 1.$
5. $-3.$
6. (i.) $\frac{3}{4}$; (ii.) $x = 5\frac{5}{106}, y = 2\frac{9}{106}.$
8. (i.) $3(3 \pm \sqrt{17})$; (ii.) $\pm 6\sqrt{\frac{a+1}{4a^2-4a-26}}.$
9. 1254.
10. $n(n-1)(n-2)(n-3).$
12. 519'78.

III. TRIGONOMETRY.

3. $\frac{8}{15}.$
5. Two.
9. $131^\circ 4' 18'' \cdot 6.$
10. $52'2405$ ft.
11. 96 cub. in., 138'528 sq. in.
12. $2\frac{1}{3}\frac{1}{8}$ tons.

IV. STATICS AND DYNAMICS.

1. 0.
2. 5 : 2.
3. $\sqrt{3} - 1.$
5. $45^\circ.$
6. 1 lb.
7. $\frac{3}{4}$ and $1\frac{1}{2}$ secs.
8. $2\frac{2}{3}$ miles.
9. (i.) $\sqrt{3} : 1$; (ii.) equal.
10. $32\sqrt{3}.$
11. $21\frac{9}{11}, 360$ feet.
12. $m = M.$

V. PURE MATHEMATICS (I).

2. Sides $7x + y = 11, 3x - y + 1 = 0, x + 3y + 7 = 0$;
bisectors $99x + 77y + 71 = 0, 7x - 9y = 37.$
3. A straight line.
4. (i.) A circle; (ii.) an ellipse, parabola, or hyperbola according as the
constant is $>, =,$ or $<$ the distance between the centres.
5. Two.
6. When the distance of the point from the origin $> \frac{OS^2}{BC}$
8. 8, $2\sqrt{3}.$
9. A parabola.

VI. PURE MATHEMATICS (2).

1. (i.) 6, -7; (ii.) $x = \pm a, \pm \frac{2a-b}{\sqrt{3}}; y = \pm b, \pm \frac{a-2b}{\sqrt{3}}$.
2. $\frac{1}{6}n(n+1)(n+2)$. 5. 89.
6. $\sin(A+2B)$ is positive, $\cos(A+2B)$ is negative.
8. $\frac{a \sin \gamma}{\sin(\alpha+\beta)}$. 9. 9'65146, 20'5309.

VII. MECHANICS.

1. 5'632, 4'792. 2. The c.g. is 3"·19 from A ; 12 oz., 17 oz.
 3. Tension, $\frac{5}{6}W$. Pressures on the pegs, $\frac{\sqrt{5}}{3}W, \frac{2\sqrt{5}}{3}W, \frac{4}{3}W$.
 7. In $1\frac{1}{4}$ secs., 37 ft. from lower point.
 8. $\frac{P-Q}{P+Q} \cdot g \cos \theta, \frac{2PQ}{P+Q} \cos \theta$. 10. (i.) $\sqrt{5ag}$; (ii.) $\sqrt{(5a-3b)g}$.
-

ANSWERS.

JUNE, 1892.

II. ALGEBRA.

1. $a^6 - a^5b + 9ab^5$.
2. $x^3 + p^2x^2 + pqx + q^2$.
3. $2x^2 + 3x - 5$; $(2x + 5)(2x - 5)(x + 1)(x - 1)$, $(2x + 5)(x - 1)(x^2 - x - 3)$.
4. (i.) $3 \div (2 - 5x + 6x^2 - 4x^3 + x^4)$; (ii.) $a + b$; (iii.) $\frac{1}{a}$.
5. $4(a + b)$.
6. (i.) $10\frac{1}{2}$, -3 ; (ii.) $\frac{2}{3}$, 2 .
7. $c^4x^2 - 4bc^2x + 4b^2 = 0$.
8. 40 .
9. $9\frac{2}{3}$, $10\frac{1}{3}$, sum 460 ; 1 and 9 .
10. 35 , 56 .
11. 12^{th} term, $-4368a^{-6}$; middle term, 12870 .
12. 18 .

III. TRIGONOMETRY.

1. At 10 min. 40 sec. after 1 o'clock.
4. 75° , 15° .
9. 82329.3 sq. ft., 100.647 ft.
11. 1886.32 sq. ft.
12. $13\frac{1}{3}$ sq. ft.

IV. STATICS AND DYNAMICS.

1. 173.2 , 200 , 173.2 . Resultant $= 2AK$, where K is mid-point of CD .
3. 12 lbs.
4. $\frac{2}{3}$ inch.
5. a must be less than 120° .
6. $\frac{3P}{\sqrt{10}}$.
8. $72 : 245$.
10. 420 .
11. Greatest height, 25 ft.; range, $100\sqrt{3}$ ft.; velocity, $16\sqrt{21}$ f.s.; distance, $32\sqrt{19}$ ft.

V. PURE MATHEMATICS.

2. $1 - 3x + 3x^2 - 3x^4 + 3x^5 - 3x^7 + 3x^8 - \dots$ 3. 8 and 4. 10. a^2 .
 11. (i.) $x + 2y = a$, $2x - y = a$; (ii.) $4x + 3y = 0$, $3x - 4y = 0$; a right angle.
 14. $7y^2 + 24xy - 24ax - 6ay + 15a^2 = 0$. Centre, $\left(-\frac{a}{3}, a\right)$; directrix,
 $12x - 9y + 29a = 0$.

VI. MECHANICS.

1. $OB : OA$. 2. Length, 1'364 inches; tension, 63'11 oz.
 3. At B , 270'4 lbs.; at E , 375 lbs.
 4. It tilts first. 6. A parabola.
 9. Velocities, v , u ; distance apart, $a - ut$. Taking point of projection as origin, and the axes horizontal and vertical, equation to C.G. is $(2x - a)^2 = \frac{u^2}{16}y$. 11. 94587 $\frac{3}{8}$ ft.-lbs.
-

NOVEMBER, 1892.

II. ALGEBRA.

1. $8x^3 - 8bx^2 + 12abx - 27a^3$, $8a^3 + 4a^2(bc)^{\frac{1}{2}} - 2abc + (bc)^{\frac{3}{2}}$.
2. $3a^3 - 2a^2b - \frac{2}{3}ab^2 + \frac{1}{3}b^3$.
3. The ascending order is $\frac{(n-1)a}{nb}$, $\frac{na}{(n+1)b}$, $\frac{(n+1)a}{nb}$, $\frac{na}{(n-1)b}$.
4. (i.) a^{m-2} ; (ii.) $\frac{2x(x+2a)}{a(x-2a)}$; (iii.) $a^2 - ab - b^2$.
5. 1, $1 + \sqrt{2}$.
6. (i.) -5; (ii.) $x = a$, $-\frac{a}{2}$; $y = \frac{a^2}{b}$, $-\frac{2a^2}{b}$; (iii.) 14.
7. $x^2 + 2x - 24 = 0$, -14.
8. $\frac{4}{3}a - \frac{3}{2}b$, $\frac{5}{3}a - b$, $2a - \frac{1}{2}b$, $\frac{7}{3}a$, $\frac{8}{3}a + \frac{1}{2}b$; 10 terms.
9. 3360, 126. 10. $1 - 2x - 2x^2 - 4x^3 - 10x^4$. 11. 4'91956.

III. TRIGONOMETRY.

1. Cosine $\frac{3}{5}$, tangent $\frac{4}{3}$, cotangent $\frac{3}{4}$, secant $\frac{5}{3}$, cosecant $\frac{5}{4}$.
4. $2 \sin \frac{A}{2} = \sqrt{1 - \sin A} - \sqrt{1 + \sin A}$. 6. '951.
8. 29'7443. 10. 322'847 ft.
11. 190'765 sq. ft. 12. 7'432 ft.

IV. STATICS AND DYNAMICS.

2. $1\frac{7}{8}$ lbs. 4. 5 lbs.
5. Perpendicular to BC ; 9 in. from B . 6. Nearly $6\frac{3}{8}$ ft.
8. 74. 9. 62 ft.

V. PURE MATHEMATICS.

1. 1210.
2. $\frac{1}{a + \frac{1}{2a + \frac{1}{2a + \dots}}}$.
3. $\frac{1}{2}, \frac{4}{7}$.
4. 3 : 4 : 5.
10. An ellipse, centre (a, o) , semiaxes $a, 2a$.
11. $y = a, 4x - 3y + 3a = 0$.
14. $4x + y = 0$.

VI. MECHANICS.

1. 5'023 lbs.
2. 1 cwt.
3. '6248 cwt.
6. 30° to horizontal.
7. $t\{v^2 - vg \sin \alpha \cdot t + \frac{1}{2}g^2 t^2\}^{\frac{1}{2}}$.
8. $g \sin \alpha \cos \alpha, g \sin^2 \alpha$.
11. $\frac{3388 Mv^2}{45(P - Mp)}, \frac{3388 Mv^2}{45(R + Mp)}$.

ANSWERS.

JUNE, 1893.

II. ALGEBRA.

1. $(2x - 5y)(3x - 4y)$, $(x^2 - 3xy + y^2)(x^2 + 3xy + y^2)$,
 $(x - 1)(x + 1)(x^2 + x + 1)(x^2 - x + 1)$.
2. $5xy(x + y)$.
3. $x^2 - 2x + 5$.
4. (i.) 0; (ii.) 4; (iii.) $1\frac{4}{9}$.
5. (i.) $3x^2 - 7x - 2$; (ii.) $\sqrt{\frac{1}{2}} - \sqrt{\frac{1}{2}}$.
7. (i.) 7; (ii.) 7; (iii.) $x = \pm 3, \pm 2$; $y = \pm 2, \pm 3$.
8. $(a - b + c)x^2 - 2(a - c)x + a + b + c = 0$.
9. -2, 0, 2, 4, 6.
11. 8th term = $-\frac{5049}{5^8}x^7$.
12. .6309.

III. TRIGONOMETRY.

1. $57^\circ 27' 27''$, $36^\circ 08' 18''$.
2. $\frac{240}{288}$, $\frac{121}{288}$, $\frac{240}{161}$.
7. $180^\circ 24'$.
8. $37^\circ 52' 27''$.
11. 1 ft.
12. $27^\circ 7' 128$ square inches, $7^\circ 542$ cubic inches.

IV. STATICS AND DYNAMICS.

1. Resultant is $\sqrt{57}P = 7.55P$, making an $\angle \tan^{-1} \frac{4}{\sqrt{3}} = 66\frac{1}{2}^\circ$ with OB .
2. $\frac{8}{9}W$, $\frac{8}{9}W$.
3. At A $96\frac{3}{4}$ lbs., at B $53\frac{1}{4}$ lbs.
4. At mid-point of BC ; $8\frac{1}{2}$ lbs., $6\frac{3}{8}$ lbs.
6. 16 lbs., 40 lbs.
7. $x = 2016$, $y = 224$.
8. $4\frac{9}{11}$ f.s.s.
9. A is $\frac{1}{2}$ ft. away when B gets to C .

V. PURE MATHEMATICS.

2. $(1+y+xy)(1+xy+x^2y).$
3. $\frac{1}{2}\{\gamma+\delta\pm\sqrt{(\gamma+\delta)^2-4\alpha\beta}\}.$
4. $\frac{1}{8}n(n-1)(n+4).$
5. $2^{2n-1}-\frac{|2n}{2(|n|^2)}.$
9. $\frac{\sin \alpha}{1-2x \cos \alpha + x^2}.$
10. O is the focus. If OT be the perpendicular from O on the tangent and OT be produced to T' so that $OT'=2OT$, a line through T' making angle α with the tangent is the directrix.
13. $\frac{a^2-b^2}{a^2} \cdot x', \quad \frac{b^2-a^2}{b^2} \cdot y'.$
14. $Ax+By=0; Bx+Cy=0.$ If these lines coincide $B^2=AC$, and the curve becomes two coincident parallel lines.

VI. MECHANICS.

1. 3.6 lbs., $\frac{8}{3}BC$ from $B.$
2. 57.7 lbs.
3. $\frac{AB}{2}(\cot \phi \tan \alpha - 1).$
4. 21.
6. S.W. wind, whose speed equals that of vessel.
10. $37\frac{1}{2}$ miles an hour; $6\frac{5}{12}$ min., $2\frac{1}{12}$ miles.

NOVEMBER, 1893.

II. ALGEBRA.

1. $x^2 + 3y^2$.
2. $x(x-4)(4x-7)$, $(y+3)(y-3)(y^2+20)$,
 $(a^2+3b^2)(a^2-3ab+3b^2)(a^2+3ab+3b^2)$.
3. $5x^2 - 6x - 7$.
4. (i.) 0; (ii.) $\frac{a}{b} - 1 - \frac{b}{a}$; (iii.) $\frac{3}{5}$.
5. (i.) $2x^2 + 3x - 5$; (ii.) $6 - \sqrt{5}$.
6. (i.) $\frac{a^2+ab+b^2}{a+b}$, $\frac{ab}{a+b}$; (ii.) 0, 7, $-2\frac{1}{2}$; (iii.) $\frac{2}{3}$, $\frac{1}{3}$.
11. The 6th and 7th terms each = $\frac{6188}{243}$.

III. TRIGONOMETRY.

1. 1'309 ft.
2. -7 , $-\frac{1}{3}\frac{7}{1}$.
3. (i.) $2\cos^3 A$; (ii.) $\sqrt{2}\tan\theta$.
8. 2'52982.
9. $\frac{1}{2}b^2\sin 2A$.
11. $151\frac{83}{112}$ cubic feet.
12. $382674\frac{2}{7}$ square miles.

IV. STATICS AND DYNAMICS.

2. $45^\circ 49' 21''$.
4. $-\frac{P}{2}$, $\frac{P}{\sqrt{2}}$, $\frac{P}{2}$.
6. $19\frac{5}{8}$.
7. After $\frac{5}{8}$ secs. and $2\frac{1}{2}$ secs. At height of $29\frac{1}{8}\frac{3}{4}$ ft.
8. $\frac{1}{80}$ ounce-weight.
9. $\frac{Qg}{P+Q}$, $\frac{PQ}{2(P+Q)}$.

V. PURE MATHEMATICS.

2. $\frac{30}{n}\{\sqrt{1+4n^2}+1\}$, $\frac{30}{n}\{\sqrt{1+4n^2}-1\}$.
3. $(1+x^n)^{-1}\{(1+x^n)^{\frac{1}{n}}-x\}^{-1}$, $\{(1+x^n)^{\frac{1}{n}}-x\}^{-1}$.
5. $2\log_a e \left\{ \frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \dots \right\}$.

6. $90^\circ, 30^\circ$.
 7. $\frac{1}{4}\sqrt{\left\{\frac{s_1}{3}(6s_1s_2-8s_3-s_1^3)\right\}}$, 6.
 8. $3\frac{3}{4}$ ft.
 9. $2x - \sqrt{6y} + 1 = 0$.
 11. $x^2 + y^2 - 3x + 2 = 0$, $2x^2 + 2y^2 - 5x - \sqrt{3}y + 3 = 0$,
 $2x^2 + 2y^2 - 7x + \sqrt{3}y + 6 = 0$.
 12. $\frac{a^2}{x'}$, $\frac{b^2}{y'}$.

VI. MECHANICS.

2. $45\frac{1}{2}$ lbs.
 3. $5\frac{1}{10}$ tons.
 5. $\tan^{-1}2\mu$.
 7. The focus is a point $30\frac{1}{2}$ ft. below the point from which the ball was dropped; the directrix is a line parallel to the direction of the train $30\frac{1}{2}$ ft. above the roof.
 8. (i.) 1200 lbs. ; (ii.) 34800 lbs.
 9. (i.) $1:3$; (ii.) $1:2e+1$.
 10. $5\frac{5}{19}$ inches.
-

ANSWERS.

JUNE, 1894.

II. ARITHMETIC.

- | | | |
|--------------------------------------|---------------------------|---------------------------------|
| 1. $22\frac{5}{7}$. | 2. .0016. | 3. £74. 11s. $10\frac{1}{2}d$. |
| 4. £1. 13s. $10\frac{1}{2}d$. | 5. $\frac{7}{40}$. | 6. £30. 12s. 3d. |
| 7. 660. | 8. $2\frac{1}{2}$ inches. | 9. 12. |
| 11. A had originally £1380, B £3220. | 12. 6 days. | |
| 13. 15. | 14. £5. | |

III. ALGEBRA.

- | | |
|--|--|
| 1. $x^3 - 3x^2 + 11x - 8$. | |
| 2. $(x - 2y)(x^2 + 2xy + 4y^2)$, $(x - 8y)(x - 16y)$, $(x^2 + xy + y^2)(x^2 - xy + y^2)$. | |
| 3. $x^2 - 7x + 2$. | 4. $5(a^2 + b^2 + c^2 - bc - ac - ab)$. |
| 5. (i.) $n^2 + 3n + 1$; (ii.) $\sqrt{13} + \sqrt{11}$. | 6. (i.) $\frac{a+c}{2}$; (ii.) 4, -7. |
| 7. $x^2 + 3px + 2p^2 + q = 0$. | 8. 16 and 40 years. |
| 9. 12, $\frac{1}{12}$. | 12. 2·5854607, ·1602526, 5·82. |

IV. TRIGONOMETRY.

- | | |
|--|---|
| 1. $343^\circ 38' 10\frac{1}{11}''$. | 2. $-\frac{2}{3\sqrt{7}}$. |
| 3. $49^\circ 177'$. | 4. $-\frac{1}{\sqrt{2}}$, $-\frac{\sqrt{3}}{2}$, -1, $\frac{2}{\sqrt{3}}$. |
| 8. $21^\circ 47' 12\frac{1}{2}''$, $38^\circ 12' 47\frac{1}{2}''$, 120° . | |
| 9. $\left\{ \frac{x^2 \sin^2 \alpha}{\sin^2(\beta - \alpha)} - \frac{2xy \sin \alpha \cos \beta \sin \gamma}{\sin(\beta - \alpha) \sin(\gamma - \beta)} + \frac{y^2 \sin^2 \gamma}{\sin^2(\gamma - \beta)} \right\}^{\frac{1}{2}}$. | |
| 10. 60° , $44^\circ 49' 20''$. | 11. 114582 sq. ft. |
| 12. 4·0825 inches. | |

WOOLWICH ENTRANCE EXAMINATION.

V. STATICS AND DYNAMICS.

1. (i.) Equilibrium impossible ; (ii.) Parallel to sides of a right-angled triangle.
2. $\sqrt{\{1440 + 304\sqrt{2} + 180\sqrt{3} - 144\sqrt{6}\}}$, $\tan^{-1} \frac{29 + 8\sqrt{2}}{10 + 9\sqrt{3} - 8\sqrt{2}}$ with $O.A.$
3. (i.) $216\frac{2}{3}$ lbs. (ii.) 200 lbs.
5. 5 lbs.-weight at distance 16" from greater force.
6. $\frac{2}{3}\frac{5}{8}$ inch from centre. 8. Ratio is 11 : 10.
9. At height 3520 ft., 11 secs. after the first was fired.
10. 3 lbs.-weight, or 96 poundals.
11. Range 7500, velocity $20\sqrt{241}$ in direction $\tan^{-1} \frac{4}{15}$ to horizontal.

VI. PURE MATHEMATICS.

2. .000762588, .000000566236. 3. $\frac{2}{3}\frac{5}{7}$ secs., $1\frac{2}{3}$ secs.
9. $\frac{x}{p} + \frac{y}{q} = 2$.
10. If the equation to the circle be $x^2 + y^2 + Ax + By + C = 0$,
 $(A + B)^2 > 8C$.
11. $x^2 + y^2 - 5ax = 0$; $\left(a, 2a\right)$, $\left(\frac{9a}{2}, -\frac{3a}{2}\right)$, and $\left(a, -2a\right)$, $\left(\frac{9a}{2}, \frac{3a}{2}\right)$.
13. $x^2 + y^2 + 2xy \frac{a^2 - b^2}{a^2 + b^2} = a^2 - b^2$.

VII. MECHANICS.

2. $n\pi$. 3. $\frac{3\delta}{\sqrt{2}}$. 5. $\frac{3}{4} l$. from end of the bar.
 8. 20,000 ft., 25.5 ft. 9. $\frac{2}{3} W$.
-

NOVEMBER, 1894.

II. ARITHMETIC.

- | | | |
|----------------------------|---------------------------|---------------------------------|
| 1. $\frac{1}{8}$. | 2. 440'82. | 3. $4\frac{1}{2}$ per cent. |
| 4. 4 cwt. 1 qr. 14 lbs. | 5. $\frac{1}{10}$. | 6. £200. 10s. $2\frac{3}{4}$ d. |
| 7. 47. | 9. £80. | 10. 3 p.m. |
| 12. Latter. | 13. £194. 5s., £224. 14s. | |
| 14. $37\frac{1}{2}$ miles. | | |

III. ALGEBRA.

- | | |
|--|--|
| 1. $x^4 + ax^3 - a^3x - a^4$. | 2. $-\frac{(x+y-z)^2}{2yz}$. |
| 4. $-(a+b+c)$. | 5. $x = -3$, $y = 1\frac{1}{2}$, $z = 4$. |
| 6. $-\frac{1}{2}$, ± 3 . | 7. -1 , $-\frac{1}{2}$. |
| 8. Series is $3 + 2\frac{1}{2} + 2 + 1\frac{1}{2} + \dots$. | |
| 9. $1 + \frac{1}{3}x + \frac{2}{3}x^2 + \frac{1}{3}x^3 + \dots + \frac{1 \cdot 4 \cdot 7 \dots (3n-5)}{3^{n-1} \cdot n} x^{n-1} + \dots$ | |
| 10. 45 and 36 miles an hour. | 11. $3'2101493$, $2'30258$. |

IV. TRIGONOMETRY.

- | | | |
|---|------------------------------|--------------------------|
| 1. $57\frac{3}{11}^\circ$. | 3. 15° . | 5. 20° . |
| 8. $B = 54^\circ 56' 48''$, $C = 83^\circ 3' 12''$, $c = 209'174$; or $B = 125^\circ 3' 12''$,
$C = 12^\circ 56' 48''$, $c = 47'2108$. | | |
| 9. $109^\circ 6'$. | 10. $3\frac{1}{7}$ sq. feet. | 11. $12'143$ sq. inches. |

V. STATICS AND DYNAMICS.

- | | |
|--|---|
| 1. $2AC$. | 3. 100 lbs., 40 lbs.; 112 lbs., 28 lbs. |
| 5. $\frac{W}{2}$, $\frac{W}{\sqrt{3}}$. | 6. 1. |
| 7. 1024 ft., 16 seconds; 4 and 12 seconds. | |
| 9. 45° , 160 f.s. | 10. 2 lbs.-weight, or 64 poundals. |

VI. PURE MATHEMATICS.

4. 103923, percentage 7725. 5. 4.
 6. $82^{\circ} 25' 22''$ and $57^{\circ} 34' 38''$.
 7. $\frac{4}{\rho} \sin \frac{\phi'' - \phi'}{2} \sin \frac{\phi'' - \phi}{2} \sin \frac{\phi' - \phi}{2}$, '000227633. 10. $y - k = \sqrt{3}(x - h)$.
 11. $2 \sin^{-1} \frac{\rho}{\sqrt{\alpha^2 + \beta^2}}$ 14. $(\sqrt{2} - 1)x^2 - (\sqrt{2} + 1)y^2 = \frac{25}{2\sqrt{2}}$.

VII. MECHANICS.

1. 7.89. 2. $\frac{W\sqrt{3}}{2}$ making angle 60° with horizon.
 3. $\sqrt{13} : 2$. 5. 14 lbs. 8. 100 ft.
 9. $3\frac{1}{2}$ seconds. 10. $50\sqrt{2}$ f.s., $\tan^{-1} \frac{4}{3}$.
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